MASARYK UNIVERSITY
Faculty of Medicine

OVERVIEW OF PHYSICS

Minimum knowledge required for entrance test – a guide to English physical terminology

Compiled and Edited by

Vojtěch Mornstein
Professor of Medical Biophysics,
Head of the Department of Biophysics,
Faculty of Medicine, Masaryk University,
Brno, Czech Republic

Second revised edition

BRNO 2010

Written by

Prof. RNDr. Vojtěch Mornstein, CSc.
Department of Biophysics,
Faculty of Medicine, Masaryk University,
Brno, Czech Republic

Revised by

Assoc. Prof. RNDr. Zdeněk Bochníček, Dr.
Department of Physical Electronics
Faculty of Science
Masaryk University, Brno
Czech Republic

This textbook has not been subjected to language revision.

ISBN 978-80-210-5192-8
ISBN 80-210-1560-8 (1. vyd.)

CONTENTS

AUTHOR'S FOREWORD

Physics is one of the sciences which provide the fundamental knowledge necessary for medical students to understand the true nature and complexity of medicine. It permits an insight into the processes taking place in the living organism, such as energy transformation and structure formation, and into the principles on which diagnostic and therapeutic devices are based. Today, it is difficult to find a branch of physics which does not contribute to the development of medicine. The mechanical properties of tissues and organs are closely related to their anatomy. The physical principles of blood circulation are essential for understanding many problems in cardiology as well as internal medicine as a whole. Thermodynamics answers the question of what is the driving force of all biological processes at molecular and cellular levels. The theory of electromagnetism is necessary to understand bioelectric phenomena, and, of course, the construction and function of electronic devices used in medicine. Knowledge of optics is required to understand the properties of the human eye, as well as, principles of numerous optical devices used for examination of patients or found in biomedical research labs. Acoustics and ultrasound in particular, is nowadays encountered in nearly every field of medicine. Ionising radiation is still one of the most powerful therapeutic agents in oncology, and it is used also in sophisticated imaging methods. Knowledge of physics is essential for the safe and effective use of medical devices.

All this implies that physics is a subject with a decisive role in the selection of students for admission to a medical faculty. This fact has provided the main incentive for producing this text, which is not a complete textbook of high school physics but a review of the required knowledge of basic physics to guide students in the preparation for the entrance tests. After more than fifteen years of experience with teaching foreign students at the Medical Faculty of the Masaryk University, the topics which we have found to be most difficult for our students, such as electromagnetism, optics and quantum and nuclear physics, have been dealt in a little more in detail. However, with some exceptions, the medical applications of the respective parts of physics are not mentioned in this overview.

Not all foreign students of our Faculty are familiar with high school physics. For example, some have knowledge only in certain "classical" parts of physics like mechanics but they do not understand nuclear physics at all. Thus I believe that this brief introductory text will also be useful in providing basic physical knowledge in English to many foreign students, who are already enrolled in the Faculty and who are preparing for their "biophysical" exam. It could also be used by the Czech or Slovak students, both undergraduate and postgraduate, who may be involved in theoretically oriented biomedical studies abroad.

The author wishes to thank Mr. Carmel J. Caruana, PhD, University of Malta, for English language editing and many valuable comments.

1 INTRODUCTION

It is possible to describe physical phenomena in various ways. The most correct description, of course, is by definitions and formulas. Every physical phenomenon can be quantified by some suitable physical quantities which are characterized by their magnitude (i.e. size or numerical value) and their quality (which is represented by some unit). Today we use the world-wide accepted SI (Systéme International) units. Apart from the seven fundamental SI units, all the remaining units can be expressed by combinations of some fundamental units. Let's introduce an example:

$F = 18$ N [kgms^{-2}]

where F is the symbol of the quantity (F – force, in this handbook the symbols of quantities will be written in *italics*), number 18 is the magnitude (value) of the quantity, N is the unit of the quantity (newton), [kgms^{-2}] are the fundamental units of the quantity. The fundamental units are commonly written in square brackets.
The following table shows the **fundamental SI units**:

physical quantity	**unit**	**symbol**
length (trajectory, position or distance)	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin	K
luminous intensity	candle	cd
quantity of a substance	mole	mol

On the basis of the fundamental SI units, many **derived SI units** are defined: N (newton), J (joule), W (watt), V (volt) etc. Besides the above units it is possible to use the unit of plane angle (radian, rad) and solid angle (steradian, sr).
For practical reasons, it is also possible to use certain **subsidiary units,** for example: minute, hour, day, year (for time measurement), litre (for volume or capacity measurement), electron volt (for energy measurement) etc.
For simple expression of various magnitudes of arbitrary units (i.e. their multiples and submultiples), it is possible to use **standard prefixes** for SI units. All the allowed prefixes are introduced in following table.

prefix	**abbreviation**	**value**	**prefix**	**abbreviation**	**value**
atto	a	10^{-18}	(deca)	da	10^{1}
femto	f	10^{-15}	(hecto)	h	10^{2}
pico	p	10^{-12}	kilo	k	10^{3}
nano	n	10^{-9}	mega	M	10^{6}
micro	µ	10^{-6}	giga	G	10^{9}
milli	m	10^{-3}	tera	T	10^{12}
(centi)	c	10^{-2}	peta	P	10^{15}
(deci)	d	10^{-1}	exa	E	10^{18}

The prefixes written in brackets are allowed only in some very often used units, for example: centimetre, decilitre, decagram, and hectolitre.
All the physical quantities can be divided into two large groups: **Vectors** (quantities with defined direction) **and scalars** (quantities without defined direction). Vectors will be sometimes introduced in the following text using

the **bold** letters. Scalars will be written by standard letters.
Example:
Vectors: **F** (force), **a** (acceleration), E (electric field intensity) ...
Scalars: m (mass), q (electric charge), W (work) ...

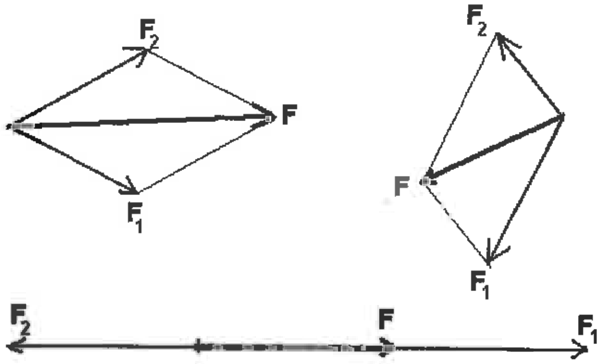

Fig. 1. Graphical addition of vectors (forces ***F***). The resultant vector is shown bold.

2 MECHANICS OF SOLID BODIES

2.1 Fundamentals of Kinematics

We will start with a description of the simplest motions (movements). This part of mechanics is called **kinematics.** Note the names of the most important quantities used in kinematics:
s [m] – displacement (trajectory, path length)
t [s] - time
$\boldsymbol{v}$ [ms^{-1}] – velocity
$\boldsymbol{a}$ [ms^{-2}] - acceleration

When studying free fall, for example, or other cases when physical bodies move in the Earth's gravitational field, the symbol $\boldsymbol{g}$ is used for the **acceleration of gravity** which has a value of 9.81 ms^{-2}.
The simplest form of a motion is the **uniform rectilinear motion** in which the vector of velocity $\boldsymbol{v}$ is constant (i.e. the magnitude, as well as, direction of the velocity remains unchanged):

$$v = \frac{\Delta \boldsymbol{s}}{\Delta t} \qquad [ms^{-1}]$$

The displacement vector $\Delta \boldsymbol{s}$ (change of body position) can be calculated according to the following formula:

$$\Delta \boldsymbol{s} = v\Delta t \qquad [m]$$

In practice, these symbols may be replaced by simpler symbols:

$$\Delta \boldsymbol{s} = \boldsymbol{s},\ \Delta t = t, \text{ and also } \Delta \boldsymbol{v} = \boldsymbol{v}$$

Uniformly accelerated motion is a motion in which the vector of acceleration $\boldsymbol{a}$ is constant:

$$a = \frac{\Delta v}{\Delta t} \qquad [\mathrm{ms^{-2}}]$$

Similarly, we can speak about **uniformly decelerated motion**, when the value of acceleration is negative. The initial velocity and position of a body will be taken as zero now ($v_0 = 0$, $s_0 = 0$). So we can express the velocity and position of the body in time t:

$$v = at \qquad \text{and} \qquad s = \frac{1}{2} a t^2$$

If the initial body position is $s_0 \neq 0$, the body position s in time t is:

$$s = s_0 + \frac{1}{2} a t^2$$

If the initial body velocity is $v_0 \neq 0$, the velocity v in time t is:

$$v = v_0 + at$$

If the initial position is $s_0 \neq 0$, and initial velocity is $v_0 \neq 0$, the position of the body s in time t is:

$$s = s_0 + v_0 t + \frac{1}{2} a t^2$$

Problem example:

A stone is thrown horizontally from a tower which height h is 50 m. Initial velocity v_0 of the stone is 40 ms^{-1}. What is duration of its fall? What will be the total horizontal distance travelled by the stone? Air friction is neglected. See Fig. 2.

Solution:

The vector of the stone velocity has two components. The horizontal component is constant. The vertical component changes from zero to the final (maximal) velocity v.

Vertical movement (it is an equivalent of free fall):

$$s = h = \frac{1}{2} g t^2 \quad \textit{hence} \quad t = \sqrt{\frac{2h}{g}}$$

Horizontal movement: $d = v_0 t$

Results: 3.19 s, 127.7 m

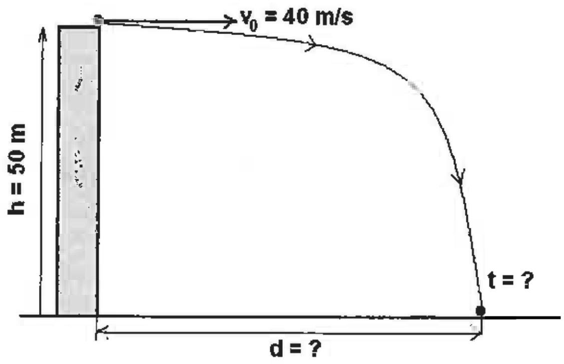

Fig. 2. Description of the problem No. 1.1.

Problems to solve:
1. On a certain planet without atmosphere, a stone falls down from a 120 m high rock in 2 s. What is the gravity acceleration there?
2. A car moves at an initial velocity of 90 km/h. The deceleration is -2.0 ms^{-2}. What time is necessary to stop the car?

2.2 Fundamentals of Dynamics

In this chapter we will briefly deal with forces changing the position and motion state of physical bodies, and energies of moving or resting bodies. In the following text, we will use the term *particle* to denote a physical object with certain value of mass but with zero dimensions.

Newton's laws of motion:
1. An object will remain at rest or in a rectilinear uniform motion unless some net (i.e. resultant) external force acts upon it.
*2. The acceleration **a** of an object is directly proportional to the net (i.e. resultant) external force F exerted upon it and inversely proportional to its mass m.*

$$\boldsymbol{a} = \frac{\boldsymbol{F}}{m} \qquad [\mathrm{ms^{-2}}]$$

3. When one body exerts a force on a second body, the second body exerts an equal but oppositely directed reaction force on the first body.
A force may be thought of as any influence which tends to change the state of motion of an object. **Newton's 2nd law** can be rewritten in this form:

$$\boldsymbol{F} = m\boldsymbol{a} \qquad [\mathrm{kgms^{-2}} = \mathrm{N} \text{ - newton}]$$

where $\boldsymbol{F}$ is the force acting or exerted, $\boldsymbol{a}$ is the acceleration and m is the mass.

Momentum $\boldsymbol{p}$ is defined as the product of velocity $\boldsymbol{v}$ and mass m of a body:

$$\boldsymbol{p} = m\boldsymbol{v} \qquad [\mathrm{kgms^{-1}}]$$

Law of momentum conservation: *When the net force acting on an isolated body or assembly of bodies is zero, the total momentum is constant.*

Impulse of a force $\boldsymbol{I}$ is defined as the product of the acting force **F** and the duration t during which the force acts:

$$\boldsymbol{I} = \boldsymbol{F}t \qquad [\mathrm{kgms^{-1}} = \mathrm{Ns}].$$

It holds:

$$\boldsymbol{I} = \boldsymbol{F}t = m\boldsymbol{v} - m\boldsymbol{v}_0 = \Delta\boldsymbol{p} \text{ (change in momentum)}$$

Impulse of a force is equal to the change of momentum that it produces.

Weight W is the force exerted by gravity:

$$W = \boldsymbol{F}_G = m\boldsymbol{g} \qquad [\mathrm{N}]$$

Density ρ is a physical quantity expressing the mass of unit volume of a substance (mass per unit volume):

$$\rho = \frac{m}{V} \qquad [\mathrm{kgm^{-3}}]$$

Torque:

Torque M (or moment of the force) is defined by the expression:

$$M = r'F\sin\alpha \qquad [\mathrm{Nm}]$$

where α is the angle between vectors r' and F. See Fig. 3. for more detailed explanation.

Any object which is at rest and remains in a fixed position is said to be in static equilibrium. A zero net external force is necessary to keep the equilibrium. *A torque may be understood as a tendency of a force to produce* ***rotation*** *about some pivot point.*

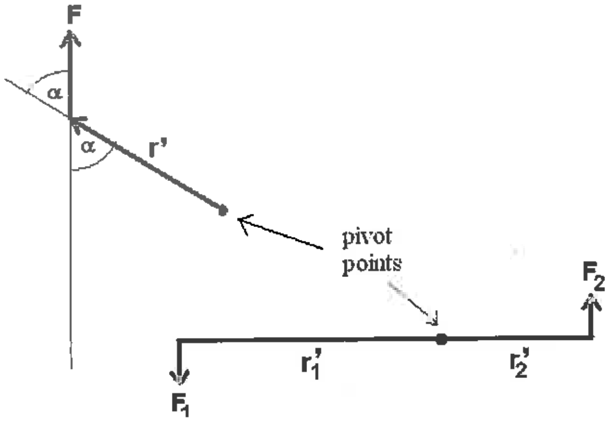

Fig. 3. Torque (moment of the force)

Torque of a couple of forces, D ($F_1 = -F_2$, see Fig. 3):

$$M = D = F_1 r_1' + F_2 r_2'$$

Friction:

Friction is a phenomenon occurring in the contact areas of two bodies, which is demonstrated as a force limiting mutual movements of the two bodies. This force, F, can be expressed in the following way:

$$F = \mu N \qquad [\mathrm{N}]$$

where F is the frictional force, μ (mu) the coefficient of friction, and N the force acting perpendicularly on the contact area (pressing the surfaces together).

We can distinguish between two different friction phenomena: **Static and kinetic friction** (static friction can be encountered in bodies which are at relative rest, kinetic friction occurs when the bodies are in relative motion).

Maximum static frictional force:

$$F_{max} = \mu_s N \qquad [\mathrm{N}]$$

Kinetic friction force:

$$F = \mu_k N$$

Uniform circular motion:

This kind of motion (when the particle or body circumscribes a circle) belongs among the uniformly accelerated motions (velocity vector is changing) – see Fig. 4.

This motion is described by time and two parameters defining the position of moving particle: The angle ϕ (phi) is called **angular displacement**, and r is the radial distance from the centre of circular motion.

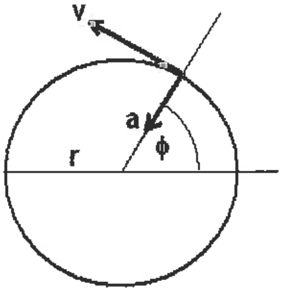

Fig. 4 – Uniform circular motion.

The rate of this motion is given by the **angular velocity** ω:

$$\omega = \frac{\phi}{t} = \frac{2\pi}{\tau},$$

where 2π is the round plane angle, t is the time during which the angular displacement ϕ (phi) is performed. τ is the time necessary for 1 complete rotation. We can also write:

$$\omega = \frac{\boldsymbol{v}}{r} \qquad [\mathrm{s}^{-1}],$$

where $\boldsymbol{v}$ is the particle velocity.

Centripetal acceleration (the vector $\boldsymbol{v}$ changes during the motion; therefore, acceleration must be defined):

$$\boldsymbol{a} = \frac{v^2}{r} = \omega^2 r \qquad [\mathrm{ms}^{-2}]$$

Centripetal force leads to centripetal acceleration:

$$\boldsymbol{F} = \boldsymbol{ma} = m\frac{v^2}{r} = m\omega^2 r \qquad [\mathrm{N}]$$

Work:

Work W is defined as the product of the force exerted on the body and the distance ***s*** *travelled by this body in the direction of the force* ***F****.*

Work – energy principle: The change of the energy of an object is equal to the work done on it by the net external force acting upon it.

$$\mathrm{W} = \mathbf{Fs}\ (\mathrm{Fs}\cos\alpha) \qquad [\mathrm{Nm} = \mathrm{J}\text{ - joule}],$$

where α is the angle made by vectors of force $\boldsymbol{F}$ and displacement $\boldsymbol{s}$. The work is a scalar.

Power:

Power P can be defined as time rate of doing work:

$$P = \frac{W}{t} \qquad [\mathrm{Js}^{-1} = \mathrm{W} - \mathrm{watt}].$$

Therefore:

$$W = Pt \qquad [\mathrm{J = Ws\ (watt\text{-}second)}]$$

Note: The often used unit watt-hour (Wh) equals 3600 J. It means that 1 kWh (kilowatt-hour) is 3.6 MJ.

Energy:

Energy E is defined as the capacity of a body for doing work. **Kinetic energy** (energy due to motion):

$$E_k = \frac{1}{2}mv^2 \qquad [\mathrm{J - joule}],$$

where m is the body mass, and v is its velocity.

Any form of energy is a scalar quantity.

Potential energy of bodies occurring in a homogeneous gravitational field (i.e. the gravitational energy) is defined by the formula:

$$E_p = mgh \qquad [\mathrm{J}]$$

where h is the "height" in which the body is positioned, m is its mass, and g is the acceleration of gravity.

The law of energy conservation in a gravitational field states that the sum of the potential and kinetic energy of a body is constant. We assume that the body is influenced only by the gravitational field. Mutual transformations of these forms of energy occur during accelerated or decelerated motion in the field of gravity, for example, in free fall etc.

Efficiency:

Efficiency η (eta) is defined as the ratio of the useful output work to the input work during a time interval.

$$\eta = \frac{W_o t}{W_i t}100 = \frac{P_o}{P_i}100 \qquad [\%],$$

where P_o is the (output) power and P_i is the input (power).

Elasticity:

Elasticity is the property of a body which tends to restore its original dimensions when the deforming stress is removed.

Normal (tensile, stretching) **stress** σ (sigma) is the restoring force F acting per unit area A:

$$\sigma = \frac{F}{A} \qquad [\mathrm{Nm^{-2} = Pa - pascal}]$$

Linear strain ε (epsilon), i.e. relative elongation:

$$\varepsilon = \frac{\Delta l}{l} \quad [\text{dimensionless}]$$

where Δl is the elongation [m] and l is the original length [m] of the body.

Hooke's law: *Until the limit of elasticity is reached, strain is directly proportional to stress.*

$$\varepsilon = \frac{\sigma}{E}$$

where E equal to the term $\frac{\sigma}{\varepsilon}$ is the **Young's coefficient (modulus) of elasticity,** an important material constant.

Problem examples:
a) The acceleration of a car weighing 1500 kg is 0.2 ms^{-2}. The respective frictional force F_f is 200 N. What is the driving force exerted by the car motor?
Solution:
Force necessary for such an acceleration of the car without friction:
$F' = ma$
The total driving force is given by the sum of the above and frictional force:
$F = F' + F_t$
Result: 500 N

b) A stone (m = 5 kg) falls down from a tower with height of 200 m. What is its velocity and kinetic energy at the moment of its impact? Air friction is neglected.
Potential energy of the stone at the top of the tower:
$E_p = mgh$
According to the law of energy conservation, we can write for kinetic energy of the stone before impact:

$$E_p = E_k = \frac{1}{2} mv^2$$

Expression for velocity:

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2E_p}{m}} = \sqrt{\frac{2mgh}{m}} = \sqrt{2gh}$$

Results: 9810 J, 62.64 $m.s^{-1}$

Problems to solve:
3. A stone is thrown perpendicularly upwards. Its initial kinetic energy is 50 000 J, its mass is 10 kg. What height will be achieved by the stone? g = 10 ms^{-2}. Air friction is neglected.
4. What is the mechanical power of a man (80 kg) going upstairs to the top of a tower (50 m) during 1 min? g = 10 ms^{-2}.
5. A steel wire 1 mm in diameter holds a body which mass is 50 kg. Young's modulus of steel is 200 GPa, acceleration of gravity 10 ms^{-2}. The length of unloaded wire was 3 m. How long is the wire now?

3 HARMONIC MOTION, WAVE MOTION AND SOUND

Harmonic motion, wave motion and sound are different kinds of periodic motion. The simplest periodic motion is called harmonic motion; it can be described by means of the sine function. The time-course of such a motion is sinusoidal.

Simple harmonic motion:
Some important terms and quantities will be defined at first. A good example of a body performing harmonic motion is a body oscillating on a spring, or a swinging pendulum. See Fig. 5.

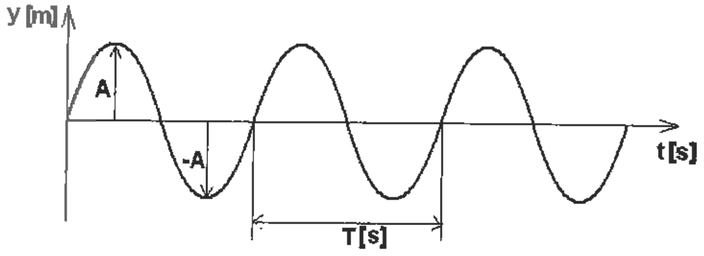

Fig. 5. Description of harmonic motion.

The **period** T [s] is the time required to complete one full cycle of harmonic motion.
The frequency

$$f = \frac{1}{T} \qquad [\mathrm{s^{-1}} = \mathrm{Hz\ -\ hertz}]$$

is the number of full cycles performed per second.

The following formulas define the instantaneous **displacement** y, **velocity** v and **acceleration** a of a harmonically oscillating particle (we do not consider their vector character here):

$$y = A\sin(2\pi ft + \varphi) = A\sin(\omega t + \varphi)$$

$$v = \omega A\cos(\omega t + \varphi)$$

$$a = -\omega^2 A\sin(\omega t + \varphi) = -\omega^2 y$$

where A is the **amplitude** of displacement [m], i.e. the maximal achievable displacement, f is the frequency [Hz], ω the **angular frequency** [$\mathrm{s^{-1}}$], t the time interval [s] elapsed from beginning of motion, and φ the **phase angle** (initial angular displacement).
Frequency and period of an **oscillating body attached to a spring** (performing free oscillations) can be calculated according to the following formulas:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad [\mathrm{Hz}] \qquad T = 2\pi\sqrt{\frac{m}{k}} \quad [\mathrm{s}]$$

where k is the so-called **stiffness constant** of the spring, and m is the mass of the oscillating body.
The frequency and period of a freely **oscillating pendulum** is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \quad [\mathrm{Hz}] \qquad T = 2\pi\sqrt{\frac{l}{g}} \quad [\mathrm{s}]$$

where l is the length of the "cord" [m], and g is the acceleration of gravity [$\mathrm{m \cdot s^{-2}}$].
Forced oscillations occur when the oscillations of a body are transferred to another body. The frequency of the forced oscillations is **resonant** when the frequency of free oscillation and the frequency of the forcing oscillation are the same.

Wave motion (notes):
The velocity of wave propagation (called also **phase velocity**) is

$$v = \frac{\lambda}{T} \qquad [\mathrm{ms^{-1}}],$$

where λ (lambda) is the wavelength [m]. We can write:

$$\lambda = vT = \frac{v}{f}.$$

Travelling (progressive) waves are characterized by changing positions of amplitudes, and they propagate at the phase velocity. Example: sound waves.

Standing waves have nodes and antinodes, they arise due to interference, positions of their maximal amplitudes (antinodes) and zero amplitudes (nodes) do not change in time. Example: an oscillating string.

In **transverse (transversal) waves**, the displacements of oscillating particles are perpendicular to the direction of propagation. In **longitudinal waves**, the displacements are of the same direction. Consider a transverse wave propagating along a rubber cord, and a train of sound wave, respectively.

We can speak about **destructive and constructive interference**. In case of destructive interference, the two interfering wave motions are subtracted; so that the resultant displacement is equal to the difference of original displacements (can be even of zero magnitude). In the second case, the displacements are added; resultant displacement is given by the sum of individual displacements of both interfering wave motions. When there is interference of two oscillatory motions of similar frequency, **beats** can arise, i.e. the resultant amplitude changes with a frequency substantially lower than the original frequency.

All real oscillations in materials **are damped**, it means that the oscillatory motion is braked, e.g. due to friction.

Intensity of wave motion (radiation) allows assessing the amount of energy which is transferred by oscillatory motion. It is defined by the formula:

$$I = \frac{P}{A} \qquad [\mathrm{Wm^{-2}}],$$

where P is power [W] and A [m^2] is the area perpendicular to the direction of wave propagation passed through by the waves. Consider sound waves passing through a frame positioned at right angles to the direction of sound propagation, which area is 1 m^2.

Sound:

Sound is a pressure wave train that can be heard.

Sound waves are longitudinal waves, but in some cases (rigid bodies) they can also by transverse.

The **pitch** of the sound is the frequency of the sound.

Infrasounds are oscillations which frequency ranges up to 16 Hz.

Audible sound is characterized by the frequency range of 16 to 20 000 Hz.

Ultrasound frequency is higher than 20 kHz.

Intensity of sound I is given in Wm^{-2}, however, e.g. in medicine, we can encounter also the quantity called "level of sound intensity" or simply **sound level** which is defined by the formula:

$$\mathrm{L} = 10\log\frac{I}{I_0} \qquad [\text{decibel - dB - dimensionless}]$$

where I is the intensity of the respective (compared) sound, and I_0 is considered to be the lowest audible intensity - **threshold of hearing** of 1000 Hz sound, which value is 10^{-12} Wm^{-2}. **Pain threshold** intensity is about 1 – 10 Wm^{-2}, the respective intensity levels are 120 – 130 dB.

Problem example:

A pendulum cord will be lengthened from "1" to "2", i.e. by 100 %. What change of frequency of the pendulum free oscillations we can expect?

Solution:
The frequency of pendulum oscillations is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}.$$

We can express the length of the cord:

$$l = \frac{g}{4\pi^2 f^2}.$$

Thus, we can write:

$$\frac{2g}{4\pi^2 f^2} = \frac{g}{4\pi^2 f'^2},$$

where f' *is the frequency of the pendulum with the longer cord. The equation can be simplified:*

$$\frac{2}{f^2} = \frac{1}{f'^2} \quad \Rightarrow \quad f' = \frac{f}{\sqrt{2}},$$

which is the result. The frequency of the "longer" pendulum is lower.

Problems to solve:
6. Explain the fact that in harmonic motion the positive displacements are characterized by negative values of acceleration and vice versa.
7. What is the sound level value of a sound which intensity is 10^{-5} *Wm*$^{-2}$*?*

4 NEWTON'S LAW OF UNIVERSAL GRAVITATION

Every particle in the Universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F = G\frac{m_1 m_2}{r^2}$$

where F is the gravitational force [N], G is the **gravitational constant** (6.66×10^{-11} Nm^{2}kg^{-2}), m is the mass [kg], and r is the distance between the bodies (particles).
$\boldsymbol{g}$ (acceleration due to gravity, acceleration of the free fall) is equal to the intensity of gravitational field $\boldsymbol{K}$ at a point (force acting on unit mass):

$$\boldsymbol{K} = \boldsymbol{g} = \frac{\boldsymbol{F}}{m} = \kappa\frac{M}{r^2} \qquad [\mathrm{Nkg^{-1} = ms^{-2}}],$$

where M is the Earth's mass and r the distance of the respective point from the centre of the Earth.
The **potential of the gravitational field** ϕ_g is the ratio of gravitation potential energy E_p of a body at a point and the mass of that body. When

$$E_p = mgh \qquad \text{then} \qquad \phi_g = \frac{E_p}{m} = gh,$$

where h is the perpendicular distance of the body from the Earth surface. The last two formulas are valid only for a homogeneous gravitational field.

Problem example:
At a certain velocity a body will circumscribe a stable circular path around the Earth. The circular path should be 5000 km above the Earth's surface. Calculate the necessary velocity. Radius of the Earth is about 6378 km, its mass equals about 5.97×10²⁴ kg.
Solution:
Centripetal acceleration and centripetal force:

$$a = \frac{v^2}{r} \text{ thus } F = ma = m\frac{v^2}{r}$$

Gravitational force (Newton's law of universal gravitation):

$$F = \kappa \frac{m_1 m_2}{r^2}$$

These forces are equal. Thus:

$$m_1 \frac{v^2}{r} = \kappa \frac{m_1 m_2}{r^2} = \kappa \frac{m_1 M}{r^2}$$

This equation can be simplified and rewritten:

$$v = \sqrt{\frac{\kappa M}{r}}$$

For r we have to substitute the sum of the Earth's radius and the distance of the body form Earth's surface. (6378 + 5000 km = 1.16378×10⁷ m). Result: 5844 ms⁻¹.

5 MECHANICS OF FLUIDS

Fluids are liquid or gaseous. In this chapter we will deal mainly with liquids, but it is necessary to keep in mind that most of the following laws and formulas are valid also for gases. To simplify the description of liquid behaviour, we introduce the term **incompressible liquid**. Volume of an incompressible liquid is assumed constant. The particles forming a liquid are held together by **cohesive forces** which are of electric origin (there are attraction forces between permanent or induced electric dipoles). The cohesive forces cause various phenomena, the most important of which is the **viscosity** due to internal friction. Liquids (fluids) without internal friction are called **ideal or perfect liquids** (fluids).
Before we start the description of liquids in motion, it is necessary to mention basic laws dealing with pressure.

Pressure:
Pressure is defined as force acting perpendicularly per unit area:

$$p = \frac{F}{A} \quad [\mathrm{Nm^{-2}} = \mathrm{Pa} \text{ - pascal}] \text{ (scalar)}$$

In medical sciences, instead of the unit pascal, older units are used – millimetres of mercury column or torrs (after Torricelli). It is useful to know that:

1 torr = 133.3 Pa

The liquid pressure caused by external forces (e.g. gravity) is called **hydrostatic pressure**. It is defined by the formula:

$$p = h\rho g$$

where h is height of the liquid column [m], g is the gravity acceleration and ρ is the liquid density [kgm^{-3}]. The hydrostatic pressure in a liquid does not depend upon the shape of the vessel (hydrostatic paradox).

Pascal's Principle:
Any change of pressure in an enclosed fluid at rest is transmitted undiminished to all parts of the fluid, i.e. pressure is exerted equally in all directions in a static fluid.

The **hydraulic press** is a typical application of the Pascal's Principle:
Two connected vessels are closed by pistons "1" and "2" which are of different areas. According to the Pascal's Principle, the pressure due to action of external (outer) forces is constant in all parts of the fluid. See Fig. 6. Thus:

$$p_1 = p_2$$

and

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

where A_1 and A_2 are the areas of the piston surfaces, F_1 and F_2 the forces acting on the pistons.

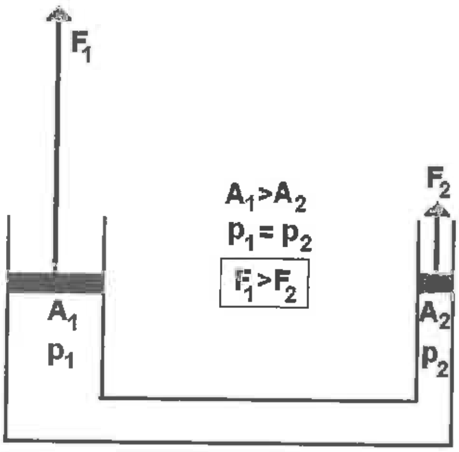

Fig. 6. The hydraulic press. A small force F_2 produces a large force F_1.

Archimedes' Principle:
A body wholly or partly submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid.

$$F = V\rho g \qquad [N],$$

where F is the buoyant force (force of buoyancy), and V is the volume of displaced fluid (i.e. the volume of the submerged part of the body).

Internal friction and viscosity:

The **internal friction** and **viscosity** are quantities describing the liquid (or fluid) ability to flow. The internal friction τ (tau) is defined by the following formula:

$$\tau = \frac{F}{A} = \eta \frac{\Delta v}{\Delta y}$$

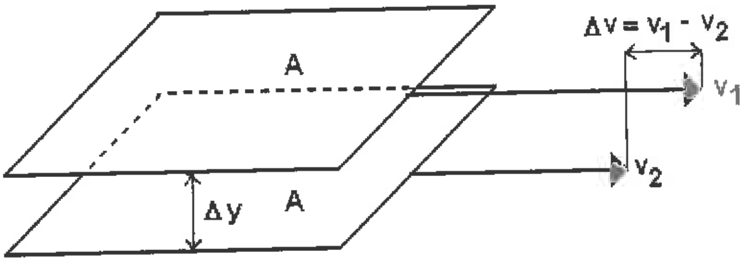

Fig. 7. To the internal friction and viscosity.

See Fig. 7. now. F is the shear force acting on two near parallel layers of a liquid. This force (parallel with the layers) causes the layers to slide over one another. η (eta) is the **coefficient of (dynamic) viscosity** [$\mathrm{Nm^{-2}s}$ = Pas (pascal-second)]. A *is* the contact area of the two layers of a liquid [$\mathrm{m^2}$], Δv is the velocity difference of the two layers, Δy is the distance between the two parallel layers.

The term $\frac{F}{A} = \tau$ defines the internal friction of the liquid.

The term $\frac{\Delta v}{\Delta y}$ is called **velocity gradient.**

The lines representing trajectories of particles forming the liquid are called streamlines. We can distinguish two basic types of flow – turbulent and laminar. The **laminar flow** is characterized by streamlines which do not intersect. The **turbulent flow** is represented by intersecting streamlines which form chaotic turbulence (whirls, vortexes).

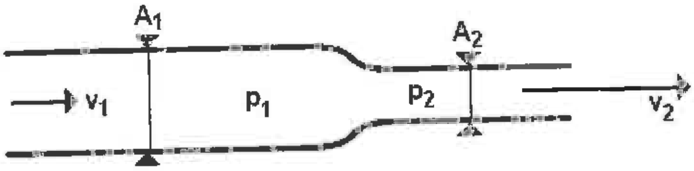

Fig. 8. A liquid flowing in a tube.

LIQUID FLOWING IN A TUBE:

The **equation of continuity** is one of the formulations of the law of mass conservation for liquids. For compressible liquids (fluids), it holds:

$$A_1 v_1 \rho_1 = A_2 v_2 \rho_2 = constant.$$

For incompressible liquids ($\rho_1 = \rho_2$):

$$A_1 v_1 = A_2 v_2 = constant,$$

where A_1, A_2 are the cross-section areas of a tube, and v_1, v_2 are the velocities of the liquid in respective parts of the tube. See also Fig. 8.

Bernoulli's equation is one of the formulations of the law of energy conservation for liquids:

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 = constant.$$

The individual terms in the equation are equal to the energy of unit volume of the liquid ($V = 1\ \mathrm{m}^3$). The symbols are explained above. Above form of this equation is valit only for horizontally oriented tubes. See also Fig. 8.

Surface tension and related phenomena:

Surface tension is another phenomenon caused by cohesive forces. It is manifested as emulsification, foaming or capillarity. The surface tension occurs at boundaries between gases and liquids, or more exactly, at **phase interfaces.**

Surface tension is the force exerted per unit length (of a virtual line lying in...) of a boundary.

$$\sigma = \frac{F}{1} = \frac{\Delta E}{\Delta A}\ [\mathrm{Nm^{-1} = Jm^{-2}}],$$

where σ (sigma) is the surface tension (coefficient) which can be also defined as the planar density of surface energy E. ΔE is a change of surface energy and ΔA is the respective change of liquid surface area.

Note also the terms **wettability, capillary elevation** and **capillary depression.**

Capillary pressure can be encountered within liquids with curved surfaces (inside drops, bubbles, under curved surfaces of liquids inside capillaries etc.) It can be calculated according to Laplace's formula:

$$p_c = \frac{2\sigma}{r} \quad or \quad p_c = \frac{4\sigma}{r}\ [\mathrm{Pa}]$$

where r is radius of liquid surface curvature. The second formula holds for bubbles in air. Under convex surfaces the liquid pressure increases by p_c, under concave surfaces – liquid pressure decreases by p_c.

Problem example:

A pipe consists of a wide and a narrow part. In the wide part, the water velocity v_1 is 2 ms^{-1}, the water velocity v_2 in the narrower part is 10 ms^{-1}. The diameter of the wide part is 5 cm. What is the diameter d_2 of the narrower part?

Solution:

Equation of continuity:

$A_1 v_1 = A_2 v_2 = constant$

Cross-section area of the pipe:

$A = \pi r^2$

Radius of the tube (one half of the diameter):

$$r = \sqrt{\frac{A}{\pi}}$$

Diameter d_2 ($2r_2$) of the narrow part of tube:

$$d_2 = 2r_2 = 2\sqrt{\frac{A_2}{\pi}} = 2\sqrt{\frac{A_1 v_1}{\pi v_2}} = 2\sqrt{\frac{r_1^2 v_1}{v_2}} = \sqrt{\frac{d_1^2 v_1}{v_2}}$$

Result: $d_2 = 2.236\ cm$

Problems to solve:

8. The force exerted by the working part of a hydraulic press is 500 kN. The driving force is only 2 000 N. The diameter of the working part (i.e. of the bigger piston) is 3 m. What is the diameter of the piston (the smaller one) which is moved by the driving force?

9. What is the force necessary for pulling upwards a thin 50 mm long metallic frame from the water level? The frame is oriented parallel to the water surface. The surface tension of the water is about 0.077 Nm^{-1}. (Note: The force of surface tension must be overcome on both sides of the frame.)

10. Calculate the pressure increase inside an air bubble of radius is 1 μm in water. The surface tension of the water is about 0.077 Nm^{-1}.

6 PRINCIPLES OF THERMODYNAMICS

Thermodynamics is a part of physics which deals with transformations of energy in thermodynamic systems. These systems are formed by a huge number of particles (atoms, molecules, etc.). In classical thermodynamics the systems are considered to be a **continuum**, i.e. matter without particular inner structure. The main forms of energy, which are involved in the thermodynamic processes, are called heat energy, electrical energy and chemical energy.

Isolated systems cannot exchange energy and the particles with their environment. Open ones can do so.

A thermodynamic system can be in two quite different states. The first one is called the **equilibrium state**. In such a state, no macroscopic processes can be observed in the system, i.e. no movement of matter or changes of energy. These processes can be observed in the **non-equilibrium states**. The latter cannot be fully described by common physical quantities such as pressure, concentration of substance or temperature because these quantities are different in individual parts of the system (or even not defined in them). The set of physical quantities which are sufficient for a full description of the equilibrium state are called **state quantities (functions, variables).**

Reversible thermodynamic processes are formed by a sequence of equilibrium states. This sequence could be "reversed", and then the same process would proceed backwardly. **Irreversible processes** are sequences of non-equilibrium states, and they cannot be "reversed".

The simplest thermodynamic system is the **ideal** or **perfect gas.** An ideal gas can be regarded as a gas formed by particles without dimensions, which do not distantly interact but are able to collide (like small rigid balls).

Temperature:

One of the most important quantities used in thermodynamics is **temperature.** It is measured using **centigrade (Celsius) temperature scale** or the **thermodynamic (Kelvin) scale:**

$$t = T - 273.15 \text{ [°C, degree centigrade = degree Celsius]}$$

where T [K] is the thermodynamic temperature sometimes called absolute (or Kelvin) temperature. In the USA, Great Britain and several other countries, the temperature is measured also in **degrees Fahrenheit.** Fahrenheit degree represents a smaller *temperature interval* than Celsius degree (9 Fahrenheit degrees = 5 Celsius degrees). The relation between the Celsius and Fahrenheit *scales* is

$$T_F = \frac{9}{5}T_C + 32 \qquad \text{or} \qquad \mathrm{T_C} = \frac{5}{9}T_F - 17.8$$

Pressure:

Pressure p is another very important quantity used in thermodynamics. This quantity is defined in the previous chapter. It is possible to find a relation between pressure and the velocity of particles forming an ideal gas:

$$p = \frac{1}{3} \cdot \frac{N.m.v^2}{V} \text{ [Pa]},$$

where N is the number of particles (e.g. molecules) in volume V, m is mass of one particle or molecule [kg], and v^2 is the average square of velocity of the molecules [$(\mathrm{m{\cdot}s^{-1}})^2$]. Since

$$E_k = E_k = \frac{1}{2} \cdot mv^2 \qquad \text{or} \qquad mv^2 = 2E_k \qquad \text{we can write} \qquad p = \frac{2}{3} \cdot \frac{N.E_k}{V},$$

where E_k, is the average kinetic energy of the molecules (particles).

We can also express the relation between thermodynamic temperature and **average kinetic energy of the particles** (starting from the ideal gas law which is explained further):

$$pV = \frac{2}{3} E_k N = nRT$$

For 1 mole of gas ($n = 1$, $N = N_A$ is the **Avogadro's constant** (6.022×10^{23} mol^{-1}):

$$T = \frac{2}{3} \cdot E_K \frac{N_A}{R} = \frac{2}{3} \cdot \frac{E_k}{k}$$

where $k = \dfrac{R}{N_A}$ is the Boltzmann constant ($= 1.38 \times 10^{-23}$ J.K^{-2}).

Ideal gas law and thermodynamic processes:

The variables which values determine a thermodynamic state (i.e. state variables) are mutually connected by means of state equations. One such state equation – the simplest one - is the **ideal gas law (= state equation of ideal gas):**

$$pV = nRT$$

$$\text{i.e. (for constant amount of substance } n\text{)} \qquad \frac{pV}{T} = \text{constant} \qquad \text{or} \qquad \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2},$$

where p is the gas pressure given in [Pa], V is the volume of the gas [m^3], T is the thermodynamic temperature [K], R the **universal gas constant** (= 8.32 J.K^{-1}mol^{-1}), and n is amount of substance [mol].

For numerous calculations, it is necessary to know what is the normal **molar volume** V_n (i.e. value valid for 1 mole of perfect gas) at normal values of pressure p_n and temperature T_n:

$p_n = 1.01325 \times 10^5$ Pa
$T_n = 273.15$ K
$V_n = 22.4 \times 10^{-3}$ m^{3}mol^{-1}

From the ideal gas law, it is possible to derive other laws which are valid only under some specific conditions. Processes which are described by these laws can be presented in a **pV-diagram** (Fig. 9.).

Boyle's law (Boyle – Mariotte law):

T is constant – an isothermal process takes place.

$$p_1 V_1 = p_2 V_2 = constant.$$

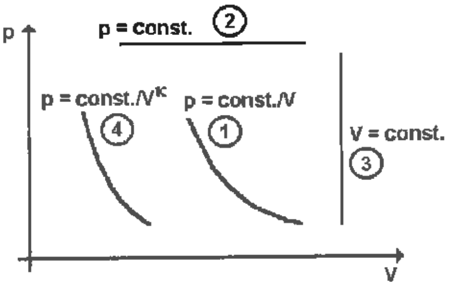

Fig. 9. The pV-diagram. 1 – isothermal, 2 – isobaric, 3 – isochoric, 4 – adiabatic processes.

Gay-Lussac's law:
p is constant – an isobaric process takes place.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = constant.$$

Charles's law:
V is constant – an isochoric or isosteric process takes place.

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} = constant.$$

Thermal expansion:
Changes in temperature lead to **expansion or contraction** of solids and liquids. **Thermal linear expansion** will be explained first:

$$\Delta l = \alpha l_0 \Delta T \qquad \text{i.e.} \qquad l = l_0 + \alpha l_0 \Delta T \ [\text{m}]$$

where Δl is the linear (longitudinal) expansion [m], ΔT is the temperature increase [K or °C], l_0 is the initial length of the body, and α is the **coefficient of linear expansion (linear thermal expansivity).**
Thermal volume expansion can be expressed in the following way:

$$\Delta V = \beta V_0 \Delta T \qquad \text{i.e.} \qquad V = V_0 + \beta V_0 \Delta T$$

$$V \cong V_0 + 3\alpha V_0 \Delta T$$

Heat and heat capacity:
Heat energy Q is the energy related to thermal motion of molecules. It is exchanged *between thermodynamic systems during thermodynamic processes.*
The unit of heat energy is joule [J]. An often used old unit of heat, the calorie (amount of the heat necessary for heating 1 g of water by 1 °C is:

1 cal (calorie) = 4.186 J

Heat capacity. Heat exchange can cause a change of body temperature. Heat capacity of a body *K* is the amount of heat causing a temperature increase of 1 °C.

$$K = \frac{Q}{\Delta T} \qquad Q = K\Delta T$$

Specific heat capacity C is the heat capacity of unit mass of a substance:

$$C = \frac{Q}{m\Delta T} \qquad Q = Cm\Delta T,$$

where C (specific heat capacity) is given in [$\text{Jkg}^{-1}\text{K}^{-1}$], m is mass of substance [kg], ΔT is the temperature change [K], and Q – heat exchanged [J].
It is necessary to know that C depends on the kind of thermodynamic process. For example, C_p – heat capacity measured in an isobaric process (at constant pressure) – is greater than C_v – heat capacity measured in an isochoric process (at constant volume).

First law of thermodynamics:
This very important physical law is an application of the energy conservation principle.

$$\Delta U = Q + W$$

where U is the **internal energy** given in [J], Q is the heat exchanged [J], and W is the work done [J]. This formula should be read in following way:
Internal energy U of a system increases when heat Q is transferred into the system, and when work W is done on the system. It decreases when heat is transferred away from the system, and by work done by the system.
Internal energy is the sum of all energies (kinetic, potential) of particles forming the system. Work can be mechanical, electrical, chemical, etc.
Mechanical work (done under constant pressure):

$$W = -p\Delta V$$

Heat transfer between two sites at different temperatures can be calculated according to the equation of **thermal conductivity** (consider a heat conducting rod):

$$Q = Q = \frac{kA\Delta Tt}{d} \qquad [\text{J}],$$

where k is the **coefficient of thermal conductivity** [$\text{JK}^{-1}\text{s}^{-1}\text{m}^{-1}$], A is cross-sectional area of the heat transferring medium [m^2], ΔT is the temperature difference between different sites of the heat transferring medium [K], t is time interval in which the heat transfer was performed [s], and d is the length of the transferring medium (length of the rod) [m].
Heat phenomena accompany also phase changes (e.g. melting, vaporization, solidification etc.).
Latent heat of fusion (melting) = latent heat of solidification
For unit mass: Specific latent heat of fusion (melting) = specific latent heat of solidification.
It is the heat required per unit mass to melt the substance without change of temperature.

$$l_f = h_f = \frac{Q}{m} \qquad [\text{Jkg}^{-1}]$$

Latent heat of vaporization = latent heat of condensation:
For unit mass: Specific latent heat of vaporization = specific latent heat of condensation.
Heat required per unit mass to vaporize the liquid without change of temperature.

$$l_v = h_v = \frac{Q}{m} \qquad [\text{Jkg}^{-1}]$$

Sublimation and desublimation, the direct changes of a solid to a gas, and vice versa, are also accompanied by consumption or liberation of heat.

Second law of thermodynamics:

Before we explain this very important law, it is necessary to introduce the **adiabatic process.** Such a process can be described by the formula:

$pV^{\kappa} = constant,$

where κ (kappa) is the ratio of heat capacities C_p/C_V.

During an adiabatic process, there is no heat exchange with the environment of the system ($Q = 0$). Most real fast thermodynamic processes are adiabatic because there is not enough time to exchange heat with the environment.

Now it is possible to describe the so-called **Carnot ideal heat engine** (in principle, it is ideal gas under a movable piston, a heat reservoir kept at temperature T_1, and a cooler kept at lower temperature T_2).

The working cycle of the Carnot engine consists of two **adiabatic** and two **isothermal reversible** processes (Fig.10).

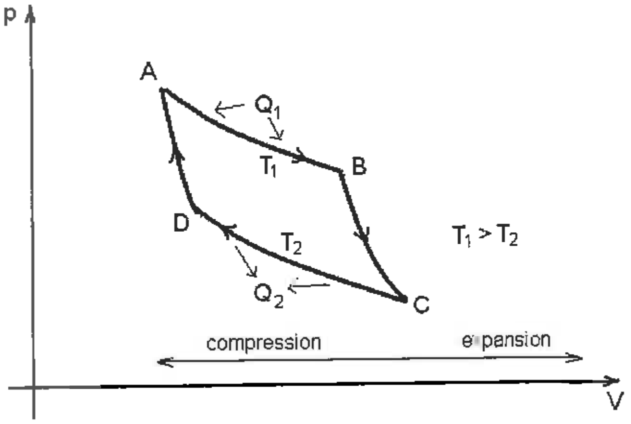

Fig. 10. The working cycle of a Carnot ideal engine.

1. Isothermal gas expansion from state A to B at T_1. Heat Q_1 is absorbed from heat reservoir. The gas does work on its environment.
2. Adiabatic gas expansion from state B to C. Temperature T_1 decreases to T_2 (no heat exchange occurs, $Q = 0$). The gas does work on its environment.
3. Isothermal gas compression from state C to D at T_2. Q_2 is transferred to the cooler. Environment does work on the gas.
4. Adiabatic compression from state D to A. Temperature T_2 increases to T_1 (no heat exchange occurs, $Q = 0$). Environment does work on the gas.

The efficiency η (eta) of a Carnot engine can be calculated according to the following formula:

$$\eta = \frac{net\ work\ done}{heat\ input} = \frac{Q_2 - Q_1}{Q_2} = \frac{T_1 - T_2}{T_1}$$

Notes:

The area closed by the curves representing the adiabatic and isothermal processes equals to the useful work done by the gas on its environment.

(Carnot's theorem) The efficiency of any heat engine operating between two specified temperatures can never exceed the efficiency of a Carnot engine operating between the same two temperatures.
Maximum achievable efficiency of this engine η could be equal to *1* (i.e. 100%) but only if the lower temperature $T_2 = 0$ K.

Statements of the Second law of thermodynamics:

- *It is impossible to transfer heat from a colder to a warmer body without doing work.*
- *Heat cannot spontaneously flow from a colder to a warmer body.*
- *It is impossible to operate a heat engine which will convert heat from a hot body entirely into work without transmitting heat to a colder body.*
- *For an isolated system, a state function called* ***entropy*** *is defined:*

$$\Delta S \geq Q/T \ [\mathrm{JK^{-1}}],$$

where Q is the heat absorbed or emitted at the temperature T.
The sign "greater than" used in the last formula is valid for irreversible processes; it means that irreversible processes taking place in isolated systems are accompanied by growth of the entropy. For adiabatic process ($Q = 0$):

$$\Delta S \geq 0$$

Therefore, the Second law of thermodynamics is also denoted as Principle of entropy increase. It can be shown that entropy is a measure of disorder – increase of entropy means increase of disorder.

Air humidity:
To a certain extent, **air humidity** and related phenomena are also a part of thermodynamics. We will define only a few basic terms:
The **absolute air humidity** is defined as mass of water vapour per unit volume of air:

$$\Phi = \frac{m}{V} \qquad [\mathrm{kgm^{-3}}]$$

where m is the mass of the water vapour [kg, more often g] in air of volume V.
Relative air humidity:

$$\phi = \frac{\Phi}{\Phi_{\max}} \cdot 100 = \frac{p}{p_{sat}} \cdot 100 \ [\%],$$

where Φ is the absolute air humidity at a given temperature, Φ_{max} is the maximum achievable absolute air humidity at a given temperature, p is the actual partial pressure of water vapour in air, and p_{sat} is the partial pressure of water vapour in air saturated by water at a given temperature.
The **dew point** is the temperature at which air is saturated by water vapour. We can measure temperature at which a cooled mirror becomes steamed up. On the basis of the dew point measurement, it is possible to measure the air humidity. The instruments used for the measurement of air humidity are called hygrometers or psychrometers.

Problems to solve:
11. What is the volume of 3 moles of perfect gas under pressure of 50 kPa? The temperature of the gas is 50°C.
12. Explain the functioning of a Carnot ideal heat engine.
13. A backwardly running Carnot engine makes a refrigerator. Why is this so?

7 THEORY OF ELECTRICITY

The theory of electricity is the part of physics dealing with properties of electric charge, the interactions between electric charges, and the conduction of electric charge in various materials.
The **electric charge** Q can be imagined as "amount of electricity" in units called coulombs (C). All electric charges are multiples of the electric charge of an electron.

$$Q_e = e = 1.602 \times 10^{-19}\ C$$

We can distinguish between two basic types of electric charge carriers and conducting media (conductors):
1st class conductors are metals in which electrons (which behaviour resembles gas) enable electrical phenomena.
2nd class conductors are electrolytes consisting of solutions of dissociated ionic compounds (e.g. NaCl). In this case, the anions and cations are the carriers of electric charge.

Coulomb's law:
This law describes the force existing between electric charges at rest. The following simple form of this law can be used only for calculations of the force exerted by "point" charges or by electrically charged spheres.

$$F = \frac{1}{4\pi\varepsilon} \cdot \frac{Q_1 Q_2}{r^2} \quad [\mathrm{N}],$$

where F is the electrostatic force, Q_1, and Q_2 are the interacting charges, r is the distance between the charges (centres of charged spheres), and ε is the **electric permittivity** of the medium between charges. We can write:

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

where ε_0 is the permittivity of vacuum ($8.85 \times 10^{-12}\ C^2 m^{-2} N^{-1}$), a very important physical constant. ε_r is the relative permittivity [dimensionless] expressing to what extent (how many times) the force acting between two electric charges is lowered in a specific medium when compared with that in vacuum.
For a vacuum, ε_r equals 1. For water, this constant reaches the value of about 81.
An electric charge can be positive or negative. A positively charged atom is due to lack of electrons, a negative one due to excess electrons. If the electric charges are of different signs (+ and -), an attractive force arises between them. A repulsive force appears when the charges are of the same sign, i.e. (- and -) or (+ and +).
All substances can be divided into three groups. These are **conductors**, which are able to transfer electric charges by free carriers which are present permanently (electrons in metals, ions in electrolyte solutions). The second group are **semiconductors** in which free charge carriers are present only under some special conditions (when increasing temperature, adding traces of certain elements, etc. – silicon is an example). The third group are **insulators**, which are not able to conduct free electric charge (e.g. glass).
An electric field produced by a charged body can cause charging of bodies in close vicinity. In conductors, we speak about **induction** of electric charge. The spatial distribution of charges is changed, unbalanced. In insulators, only **polarization** occurs. In this case, the charge carriers cannot move between individual atoms or molecules. These particles become **polar** ones, i.e. **dipoles** are formed. Of course, many microparticles are of polar character also in the absence of an external electric field (e.g. water molecules). These **permanent dipoles**, however, become oriented (arranged) according to **electric force lines** in the presence of an external electric field.

Properties of an electric field:
Proceeding from Coulomb's law, we can define very important physical quantities: electric field intensity, electric potential and potential difference (voltage). All of them characterise some properties of the electric field.

Electric field intensity (strength):

*Definition: Electric field intensity **E** can be defined as the electrostatic force acting on a unit positive charge at a particular point.*

$$\boldsymbol{E} = \frac{F}{Q} = \frac{1}{4\pi\varepsilon} \cdot \frac{Q'}{r^2}$$

[$NC^{-1} = Vm^{-1}$ (volt per meter)] (vector)

Potential and potential difference (voltage):

The electric potential at a point P is the work done in moving a unit positive charge from infinity to that point.
The potential difference between two points is the work done on a unit charge in moving it from one point to the other.

$$\phi = \frac{W}{Q} \text{ [JC}^{-1} = \text{V (volt)]} \qquad \text{(scalar)}$$

Let us have a homogeneous electric field formed by two conducting plates at a distance *d*. One plate is positive; the second one is connected with Earth. It holds then:

$$\phi = Ed$$

where ϕ is potential of the positive plate, and ***E*** is electric field intensity in the space between the plates. It can be explained as follows: The potential difference (voltage) is defined (see above):

$$U = \phi_2 - \phi_1 \text{ [V]} \qquad \text{(scalar)}$$

For the above described arrangement of plates (one positive and one connected with Earth).

$$U = \phi_2 - \phi_1 = \phi - \phi_E = \phi$$

because the Earth's potential ϕ_E is considered to be zero. It is necessary to note that in most American or British textbooks, the symbol *V* is used for voltage, instead of *U*.

Capacitance and capacitor:

Under different conditions, different amounts of electric charge can be stored in various conductors. The **electrical capacitance** (also **capacity**) *C*, i.e. amount of electric charge at unit voltage, is used for quantification of that.

$$C = \frac{Q}{U} \text{ [CV}^{-1} = \text{F (farad)] (scalar)}$$

The **parallel-plate capacitor** is formed by a couple of metallic plates. The electric capacitance of such a capacitor is given by the formula:

$$C = \varepsilon \cdot \frac{A}{d} \text{ [F]}$$

where ε is the electric permittivity of the medium between the plates [Fm^{-1}] – the same constant appears in the Coulomb's formula. *A* is the area of the plates [m^2], and *d* is the distance between the plates [m].
In various electric circuits, we can encounter two different kinds of connections of capacitors. These are:

1. Capacitors connected in parallel:

The total capacitance of such capacitors is given by the sum of individual capacitances:

$$C = C_1 + C_2 + C_3 + \ldots\ldots$$

2. Capacitors in series:
Reciprocal value of the total capacitance of such capacitors is given by the sum of reciprocal values of individual capacitances:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots$$

The energy "stored" in a charged capacitor is given by the formula:

$$E = \frac{1}{2} CU^2 \text{ [J]}.$$

Electric current and Kirchhoff's laws:
The **Electric current** I is a physical quantity used to describe a flow of electric charge.
The electric current through a wire is a measure of the amount of charge which passes through it per second.

$$I = \frac{Q}{t} \text{ [Cs}^{-1} = \text{A (ampere)]} \qquad \text{(scalar)}$$

Thus:

$$Q = I.t \text{ [C = As (ampere-second)]}$$

This formula is very useful for calculations of electric charge.
The electric current, is described by two very important **Kirchhoff's laws.**
I. Kirchhoff's law (current law – law of charge conservation – electric charge cannot be created or disappear):
The algebraic sum of the currents at any point (junction) in the circuit is zero.
II. Kirchhoff's law (so called voltage law – a special form of the law of energy conservation):
Around any loop in a network, the algebraic sum of the electromotive forces (i.e. voltages of voltage sources) and the voltage drops on resistors is zero.

Ohm's law, conductivity and resistance:
Ohm's law is a physical principle describing the relation between electric voltage and electric current in a conductor. We will describe the simplest case of a wire conductor. *The current I through a conductor is directly proportional to the potential difference (voltage) U between its ends and inversely proportional to its resistance R:*

$$I = \frac{U}{R}$$

Thus:

$$R = \frac{U}{I} \text{ [VA}^{-1} = \Omega \text{ (ohm)]} \qquad \text{(scalar)}$$

where R is the **resistance** measured in ohms [Ω].
Definition: 1 Ω is the resistance of a conductor through which a current of 1 A passes when the potential difference across the ends of the conductor is 1 V.
The resistance depends on the geometry of the conductor. Therefore, a geometry-independent quantity **resistivity (specific resistance)** was introduced:

$$R = \rho \cdot \frac{l}{A} \; [\Omega\text{m}]$$

where l is the length of a conductor [m], A is the cross-section area of the conductor [m²], and ρ (ro) is the resistivity [Ωm – ohm-meter].

Conductance G **and conductivity** κ (kappa) are the reciprocal values of resistance and resistivity, respectively.

$$G = \frac{1}{R} \; [\Omega^{-1}] = \text{S (siemens)]} \qquad \text{and} \qquad \kappa = \frac{1}{\rho} \; [\text{Sm}^{-1}]$$

Resistivity and related quantities depend on temperature:

When the temperature rises, the resistance of a metal conductor increases. This statement can be expressed by the following formula:

$$R = R_0(1 + \alpha t)$$

where R_0 is the resistance of the conductor at the temperature $t = 0$ °C, t is the temperature [°C] of the conductor, and α (alpha) is the **temperature coefficient of resistance.** At conductors of very low temperatures, a sudden decrease of electric resistivity to zero can be observed. This phenomenon is called **superconductivity**. It is exploited in some medical devices.

Resistors in electric circuits can have different arrangements. Similarly to capacitors, they can be connected in series or in parallel.

The total resistance of resistors connected in series is the sum of the resistances of individual resistors:

$$R = R_1 + R_2 + R_3 + \ldots..$$

The reciprocal value of total resistance of resistors connected in parallel is given by the sum of reciprocal resistance values of individual resistors:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots..$$

Electrical work and power:

Electrical work, power and heating effect of electric current are of great practical importance and their calculations in different devices and appliances for utilization of electric current as a heat source (for cooking, home heating etc.) is common.

Energy in transferring a charge Q between two points of potential difference U is given by:

$$W = QU = UIt \; [\text{J}]$$

where I is the current passing through the electric conductor and t is the time of its passage.

Electric power can be calculated:

$$P = \frac{W}{t} = UI = RI^2 = \frac{U^2}{R} \qquad [\text{W-watt = volt-ampere}]$$

Heat energy liberated by an electric current is called **Joule's heat**. Assuming all electrical energy is converted to heat energy, it can be expressed by the above formula:

$$W = UIt \qquad [\text{J = watt-second}]$$

The commonly used unit of electrical energy, 1 kWh (kilowatt-hour) equals 3,600,000 Ws = J (= 3.6 MJ).

Thermoelectric phenomena:
In a junction of two different metals, a small electric voltage arises due to different energy of electrons in different metals. Similarly, a small voltage can be measured across a conductor whose ends are at different temperatures. Both phenomena are called **thermoelectric phenomena**. Two junctions of metals, which are of different temperature, connected in series, form a thermocouple. Thermocouple batteries are used for production of electric energy and for simple and precise measurement of temperature.
Thermoemission is liberation of electrons from very hot metal surfaces. It is used in cathode ray tubes (CRT), X-ray tubes etc.

Electric currents in electrolytes:
The behaviour of electric charges in electrolytes is studied mainly in physical chemistry and electrochemistry. In this review of physics we will mention only **Faraday's law of electrolysis.** We will deal with transfer of electric charges (ions) between an electrolyte and an immersed metallic electrode.
Definition: The mass m of the substance liberated from an electrolyte solution is proportional to the product of the current I and the time t of current passage through the electrolyte.

$$m = AIt = AQ \qquad [\mathrm{kg}]$$

where A is the **electrochemical equivalent.** After rewriting the previous formula, we obtain:

$$A = \frac{m}{Q} \qquad [\mathrm{kgC^{-1}}]$$

it is also evident that

$$m = Nm_0 = N \cdot \frac{M_m}{N_A} \qquad and \qquad Q = NQ_0 = Nze$$

where N is the number of ions transferred between the electrolyte and the electrode, m_0 is the mass of the individual ions transferred [kg], M_m is molar mass, i.e. the mass of one mole of substance [kg] which is present in ion form, N_A is the Avogadro's constant – the number of particles in one mole of any substance, Q_0 is the electric charge of one ion [C], e is the elementary charge – 1.602×10^{-19} C, and z is the valence of the ion (ionized atom or molecule). Thus, after substitution

$$\frac{m}{Q} = \frac{M_m}{zeN_A} = \frac{M_m}{zF} = \mathrm{A} \quad \text{and} \quad \mathrm{m} = \frac{M_m It}{zF}$$

where $F = eN_A$ is **Faraday's constant (*96 500 C* –** charge of 1 mole of elementary charges).

Semiconductors:
In semiconductors, electrons can be in two different energetic states. In the basic state, the electrons are tightly bound to atoms or molecules forming the crystal lattice (they are in the valence band). However, they can be excited to a state characterized by a higher level of energy, in which they can move similarly to electrons in metals. The range of energies, which allows the electrons to move, is called the **conduction band.**
Semiconductors can be intrinsic (pure). In such semiconductors, the conduction band is filled by free (conduction) electrons and positive holes (vacancies) in the valence band. This state arises after delivery of energy (e.g. light, heat, ionizing radiation, etc.).
Extrinsic semiconductors are much more important in practice. They are produced by the addition of the impurities to some intrinsic semiconductors, e.g. to silicon. Silicon is a tetravalent element the crystal lattice of which is schematically shown in Fig. 11. and 12. Each silicon atom is connected by four electron pairs to the

surrounding atoms. A conduction band, i.e. free, movable electrons or positive holes, arises after addition of a pentavalent or trivalent impurity.

N – type semiconductors arise when adding a pentavalent impurity (e.g. arsenic, antimony, phosphorus) – see Fig. 11. In such a case, the **majority carriers** of charge become the electrons, minority carriers are the positive holes. We can say that the impurities are **donors** of electrons (forming free electrons).

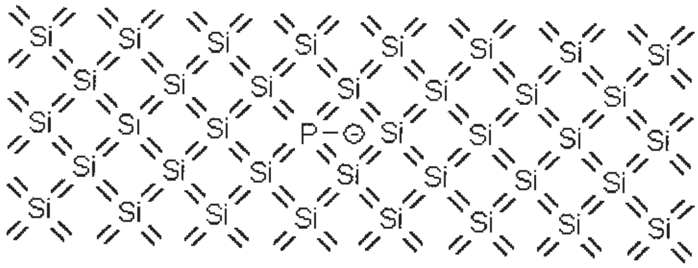

Fig. 11. Structure of an N-type semiconductor. The phosphorus atom is the impurity in the silicon crystal lattice.

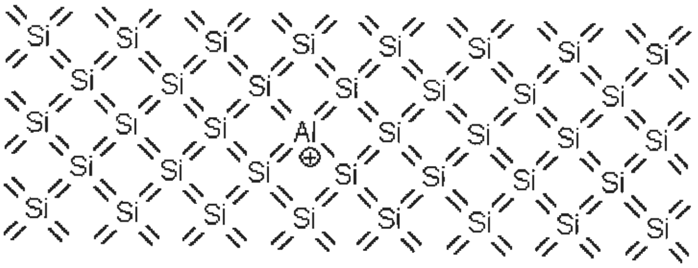

Fig. 12. Structure of a P-type semiconductor. The aluminium atom is the impurity in the silicon crystal lattice.

P – type semiconductors arise when adding a trivalent impurity (e.g. aluminium) – see Fig. 12. The majority carriers of the electric charge are now positive holes, the minority carriers are electrons. The impurities are **acceptors** of electrons (forming positive holes).

From a practical point of view, the properties of boundaries called **junctions** between P- and N-type semiconductors are very important. In such a P-N junction, the electrons diffuse from N to P, and the positive holes from P to N). This junction cannot be passed by electric current in both directions. Therefore, it can be used for rectifying alternating electric currents (as a diode). See also Fig. 13.

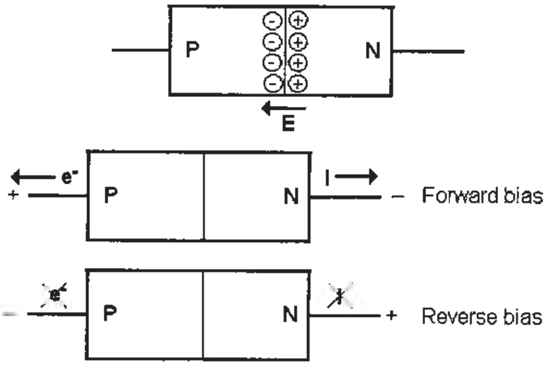

Fig. 13. The P-N junction (above, no external voltage), forward bias and reverse bias. E – electric field generated at the junction, I – electric current, e^- - electron flux

P-type semiconductor is charged positively, the N-type one negatively: The **forward bias** is formed – electric current can flow through the junction.
P-type semiconductor is charged negatively, N-type one positively. The **reverse bias** is formed – electric current cannot flow through the junction.
Principle of the **transistor** (important terms: emitter, base, collector), which is used for amplification of electric signals, can be explained also on the basis of properties of P-N junctions.

Problem example:
Three identical resistors are connected as shown in Fig. 14. Their total resistance R_t is 1 $M\Omega$. What is the resistance R of one resistor?

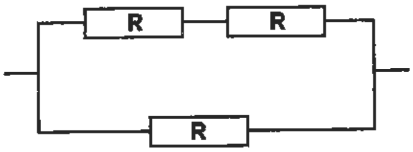

Fig. 14. Connected resistors.

$R_t = 1M\Omega \quad R = ?$
Solution: This part of an electric circuit consists of two branches. In the first branch, the resistors are connected in series, i.e. their total resistance is given by the sum of their individual resistances. Therefore, we can consider there are "two" resistors connected in parallel (R+R and R).

$$\frac{1}{R_t} = \frac{1}{R+R} + \frac{1}{R} = \frac{3}{2R}$$

Thus $R = 3/2R_t$
Result: $R = 1.5\ M\Omega$

Problems to solve:
14. An electric heater has a power of 500 W. What is the amount of heat energy produced in one hour?
15. An electric charge of 5000 C was transferred through a wire during 30 min. What was the respective value of electric current?
16. Capacitors are connected according to the Fig. 15. $C_1 = 5\ \mu F$, $C_2 = 25\ \mu F$, $C_3 = 0.1\ mF$. *What is the total capacity C of the capacitors?*

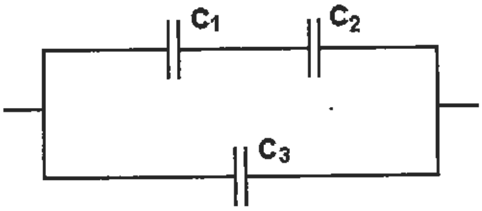

Fig. 15.

17. 5 g of copper was deposited on one electrode placed in a $CuSO_4$ *solution* (M_{Cu} = 63.5 g). *What was the amount of electric charge transferred?*

8 MAGNETISM AND ELECTROMAGNETISM

In this chapter the phenomena caused by moving electric charges will be discussed. We will start with the classical description of a magnetic field.

Force acting between two magnetic poles:
Permanent magnets (e.g. magnetised iron rods) consist of north seeking pole = north pole (N) and the south seeking pole = south pole (S). There is a well known rule describing the force acting between the two poles: "*Like poles repel – unlike poles attract.*"
The magnitude of the attraction or repulsion **force** can be calculated according to the formula

$$\boldsymbol{F} = \frac{m_1 m_2}{4\pi\mu d^2} \quad [\mathrm{N}],$$

where m_1 and m_2 is the magnetic "strength" of the poles, μ is the **magnetic permeability** [$\mathrm{Hm^{-1}}$ – henry per meter]. We can write:

$$\mu = \mu_r \mu_0$$

where μ_r is the relative permeability of the medium [dimensionless], μ_0 is the permeability of vacuum ($4\pi\times10^{-7}$ $\mathrm{H.m^{-1}}$), and d is the distance between the poles [m].
The magnetic field is described by magnetic force lines. Outside the magnet, the force lines are directed **from N-pole to S-pole.**

Magnetic flux and magnetic flux density:
The **magnetic flux** Φ passing through the area A is defined as the total number of magnetic force lines passing through an area A.
The **magnetic flux density *B*** at a point is the magnetic flux in a unit area placed at right-angles to the magnetic force lines.

$$\boldsymbol{B} = \frac{\Phi}{A} \qquad [\mathrm{Wbm^{-2}} = \mathrm{T\ (tesla)}]$$

where Φ is the magnetic flux [Wb – weber]. The magnetic flux density is sometimes called **magnetic induction** or simply *B*-vector.

Magnetising force (magnetising strength, intensity of magnetic field):
Magnetising force ***H*** is regarded as the cause of the total magnetic flux density B in the medium of permeability μ. We can write:

$$\boldsymbol{B} = \mu \boldsymbol{H} \qquad [\mathrm{T\ (tesla)}]$$

where $\mu = \mu_r\mu_0$ is magnetic permeability, ***H*** is magnetising force [$\mathrm{Am^{-1}}$ – ampere per meter]. According to the value of relative permeability, different materials can be divided into three groups:
diamagnetics: $\mu_r \leq 1$ – *B* is slightly lowered, if compared with the vacuum
paramagnetics: $\mu_r \geq 1$ – *B* is slightly increased
ferromagnetics: $\mu_r \gg 1$ – *B* is many times increased
In ferromagnetic materials, the atoms can be arranged into so-called **magnetic domains**, in which the vectors of magnetic flux density are spontaneously oriented in the same direction. During magnetisation and demagnetisation processes, these domains behave as small individual magnets. Therefore, when oriented in the same

direction during magnetisation, the magnetic fields of the domains add up – we can obtain a strong permanent magnet from originally non-magnetic material (typically steel).
The Curie temperature is the temperature at which the magnetisation of a material is removed by thermal movement (magnetic domains lose their order) – 770 °C for iron.

Magnetic field due to a straight wire or coil carrying current:
Convention: *Each moving electron produces an anti-clockwise magnetic field around itself when viewed along the direction of its motion.* See also Fig. 16(a).
A - coil or solenoid carrying current produces magnetic field and behaves similarly to a permanent magnet, N and S poles can be distinguished in them as well. Several important formulas can be derived using more complex theory of magnetism.

1. Magnetic flux density around an infinite straight wire can be calculated according the formula:

$$\boldsymbol{B} = \frac{\mu_0 I}{2\pi d} \qquad [\mathrm{T}],$$

where I [A] is the electric current, d [m] is the distance of a measurement point from the wire. See also Fig. 16a to determine the orientation of the vector $\boldsymbol{B}$.

2. Magnetic flux density in the middle of a circular coil (a simple wire loop):

$$\boldsymbol{B} = \frac{\mu_0 I}{2r} \qquad [\mathrm{T}],$$

where I [A] is the electric current, r [m] is the radius of the coil.

3. Magnetic flux density inside an infinite solenoid:

$\boldsymbol{B} = \mu_0 nI$ [T] (vacuum or air within the solenoid) and
$\boldsymbol{B} = \mu nI$ [T] (valid for a solenoid with a core of material having $\mu = \mu_r\mu_0$),

where I [A] is the electric current and n the number of solenoid turns per meter (i.e. $n = N/l$, where N is the total number of turns and l is the length of the solenoid).

Magnetic force exerted on a conductor carrying current:
If a straight wire carrying current I is positioned in a homogeneous magnetic field with magnetic flux density $\boldsymbol{B}$ making an angle α with the vector $\boldsymbol{B}$ (see Fig. 16b), the force acting on it can be calculated according to the formula:

$$\boldsymbol{F} = \boldsymbol{B}Il\sin\alpha \qquad [\mathrm{N}],$$

where l is the length of the wire (conductor) exposed to the magnetic field.
With respect to the last formula, *a magnetic flux density 1 T exists if the force exerted on a straight wire of 1 m length is 1 N when the wire carries a current of 1 A and is placed at right-angles to the direction of the magnetic flux.*
How to determine the direction of the force exerted? We can use a simple rule:
Align one of the externally applied magnetic lines of force to be parallel with and touching one of the magnetic lines of force produced by the current. The direction from this common point to the conductor gives the direction of force on the conductor. See also Fig. 16(c).

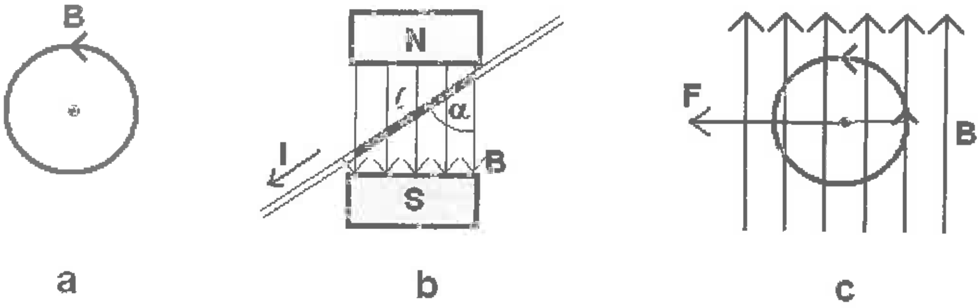

Fig. 16. (a) Vector B produced by a current carrying wire – if the current goes "from" the page, B is oriented anti-clockwise. (b) A current carrying conductor positioned in a homogeneous magnetic field. (c) A current carrying conductor positioned perpendicular to a homogeneous magnetic field – determination of the force exerted on the conductor.

Magnetic force between two parallel conductors:
The force per unit length on wire **B** (carrying current I_B) due to the magnetic field produced by the current I_A in the wire A is given by:

$$F = \frac{\mu_0 I_A I_B}{2\pi d} \qquad [\mathrm{N}],$$

where d is the perpendicular distance between the wires. If the currents have the same direction the wires are attracted. In case they have opposite direction they are repelled.

Definition of 1 ampere in SI:
A current of 1 A flows in one infinite straight wire if an equal current in a similar wire placed in parallel 1 meter away in a vacuum produces a mutual force of $\mu_0/2\pi$ N per 1 meter of length (i.e. 2×10^{-7} N.m^{-1}, since $\mu_0 = 4\pi\times10^{-7}$ N.m^{-1}).

Magnetic deflection of a moving electron:
The force exerted on an electron with charge e travelling at velocity v at an angle Θ with respect to a magnetic flux density $\boldsymbol{B}$ is given by (see also Fig. 17a):

$$F = Bev\sin\Theta \ [\mathrm{N}].$$

To determine the direction of this force use the rule shown in Fig. 16(c) because the moving electron represent also an electric current.

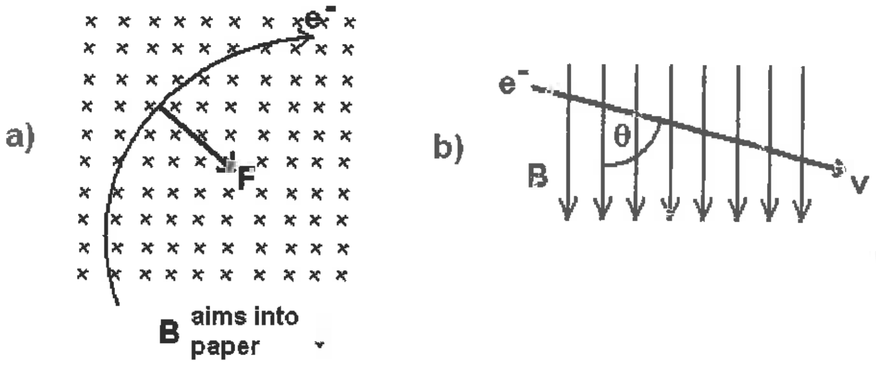

Fig. 17. Magnetic force exerted on a moving electron and meaning of the angle Θ

Electromagnetic induction:

Electromagnetic induction is the production of an electric voltage (and hence a current) in conductors due to a changing magnetic field. It is described by the **Faraday's laws:**

1. *A change of the magnetic flux linked with a conductor induces an electromotive force (voltage) in the conductor.*
2. *The magnitude of the induced electromotive force is proportional to the rate of change of magnetic flux linkage*

The "linkage" is the number of force lines intersected by the moving conductor or the product of the magnetic flux Φ and number of turns in a solenoid, etc.

For the induced voltage U we have:

$$U = -\frac{\Delta\Phi N}{\Delta t} \qquad [\mathrm{V}],$$

where $\Delta\Phi.N$ is the change of magnetic flux linkage, Δt the time of change, and N the number of turns.

Lenz's law: *The direction of the induced current in a conductor caused by a changing magnetic flux is such that its own magnetic field opposes the change in magnetic flux.*

Voltage induced in a straight wire:

Voltage induced between two ends of a conductor moving in magnetic field is given by:

$$U = \boldsymbol{B} l v \sin\alpha \qquad [\mathrm{V}],$$

where l is the length of the conductor (wire) measured in meters, v is the velocity of the conductor [m.s^{-1}], and α is the angle made by $\boldsymbol{v}$ and by the direction of the wire. We assume that the vector v and the wire are perpendicular to $\boldsymbol{B}$. If not, we have to calculate with their components lying into the plane perpendicular to $\boldsymbol{B}$ (it is the plane of this page). See also Fig. 18.

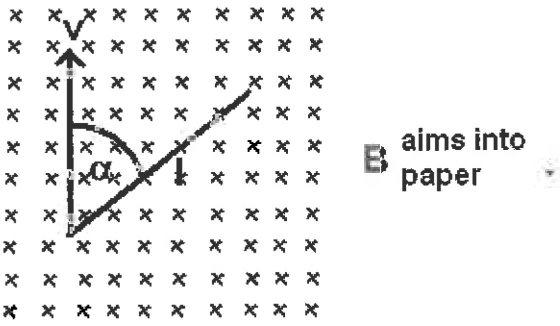

Fig. 18. A wire moving in magnetic field. l is the length of the wire (a conductor), v is its velocity.

Voltage induced by change of current in a solenoid:

It is given by the formula:

$$U = L \cdot \frac{\Delta I}{\Delta t} \qquad [\mathrm{V}]$$

where L is the **self-inductance** of the solenoid and

$$L = \mu \cdot \frac{N^2 A}{l} \qquad [\mathrm{H - henry}]$$

In the last formula, N is the number of solenoid turns, l is the length of the solenoid [m], A is the cross-sectional area of the solenoid [m^2], and μ is the permeability of the medium inside the solenoid.

Alternating current (AC):

Alternating current (AC) is used for energising various electrical devices. It is also efficient to use for long-distance delivery of electrical energy. To describe alternating currents or voltages, we can use similar quantities as used in the description of simple harmonic motion:

The **cycle** is one complete waveform.

Period T [s] is the time duration of one cycle.

Frequency $f=\frac{1}{T}$ [Hz = s^{-1}] is the number of cycles per second.

Amplitude is the maximum value of positive or negative current (or voltage). Instantaneous values of alternating current and voltage are:

$$I_t = I_p \sin(2\pi f t + \phi) \quad [\mathrm{A}] \quad \text{and}$$
$$U_t = U_p \sin(2\pi f t + \phi) \quad [\mathrm{V}],$$

where I_t and U_t are the instantaneous values of current or voltage, I_p and U_p are the amplitudes of current or voltage (their peak values). ϕ (phi) is the phase angle which defines the values of I or U in time $t = 0$.

Note: Peak-to-peak values = peak values multiplied by 2. Average value of alternating current or voltage is zero, because their variation with time is sinusoidal.

Effective (root mean square – RMS) current and voltage:

The effective voltage (current) is that value of constant direct voltage (current) which would produce the same expenditure of electrical energy in the circuit as the alternating voltage (current). The following equations hold:

$$I_{eff} = \frac{I_p}{\sqrt{2}} \quad \text{and} \quad U_{eff} = \frac{U_p}{\sqrt{2}}$$

where I_p and U_p are the amplitudes of current or voltage (their peak values).

Electrical power of alternating current:

$$P = U_{eff} I_{eff} = R I_{eff}^2 = \frac{U_{eff}^2}{R} \qquad [\mathrm{W = VA = watt = volt\text{-}ampere}]$$

The previously mentioned Ohm's law is valid for AC only when using effective values of voltage and current.

Reactance and impedance:

Reactance can be described as "resistance to the flow of electric charge in AC circuits". Impedance is the total reactance. In general, the reactance and impedance Z depend on the frequency of the alternating current.

Reactance of a **capacitor** in an AC circuit:

$$X_C = \frac{1}{2\pi f C} \qquad [\Omega],$$

where X_C is the **reactance** of a capacitor of **capacity** C [F], and f is the frequency [Hz] of the alternating current. The higher the frequency of an alternating current, the smaller is the reactance of a capacitor.

When uncharged, i.e. when the voltage across its plates equals zero, the capacitor is charged by the greatest current. This means that the voltage is delayed with respect to the current with a phase difference of $+\pi/2$, i.e. the amplitude of the current occurs by *T/4* earlier than the amplitude of the voltage.

Reactance of an **inductor (solenoid)** involved in an AC circuit:

$$X_L = \omega L = 2\pi f L \ [\Omega]$$

where X_L is the reactance of an inductor, and L is the self-inductance [H] of the inductor. The higher the frequency of an alternating current, the greater is the reactance of an inductor.

The alternating current passing through the winding of the solenoid produces a magnetic field. This field induces a voltage in the solenoid which has an opposite polarity to the voltage of the source (see Lenz's law). This means that the current is delayed with respect to voltage with a phase difference of $-\pi/2$, i.e. the amplitude of the current occurs by *T/4* later than the amplitude of voltage.

It is often necessary to calculate the impedance of complex parts of electric circuits. For example, the following formula can be used for a resistor, capacitor and inductor (RCL) in series (**Fig.** 19):

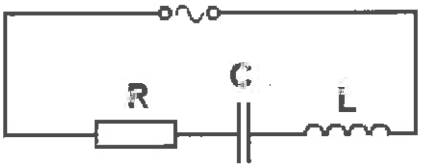

Fig. 19. A resistor (R), capacitor (C) and inductor (L) connected in series.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \ [\Omega]$$

where R is the resistance. The current in such of electric circuits is able to oscillate, which is very important for transmission of electromagnetic waves (radiofrequencies). The strongest oscillation (resonance) occurs when the impedance of the circuit is minimal. According to the last formula, the smallest Z can be reached when $(X_L - X_C)^2 = 0$, i.e. $X_L = X_C$. This condition allows determining the resonance frequency:

$$2\pi f L = \frac{1}{2\pi f C} \quad \rightarrow \quad (2\pi f)^2 = \frac{1}{LC} \quad \rightarrow \quad f = \frac{1}{2\pi\sqrt{LC}}$$

AC transformer:

This device is used for transformation of electric alternating voltages from high to low values and vice versa. A transformer consists of two solenoids with common core, i.e. primary (p, input) winding, the secondary (s, output) winding, and the soft iron core – see Fig. 20.

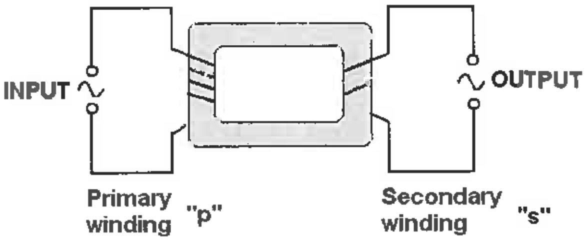

Fig. 20. The AC transformer. The iron core is drawn in grey colour.

During transformation, a part of the electric energy is lost, mainly by conversion into heat. Therefore, ideal (efficiency = 100%) and practical transformers can be distinguished. The following formulas are used for some practical calculations on transformers:

$$\frac{U_p}{U_s} = \frac{n_p}{n_s} \quad \text{and} \quad \frac{I_s}{I_p} = \frac{n_p}{n_s},$$

where U is the voltage across the winding [V], I is the current through the winding [A], and n is the number of turns in the winding. Thus, in the transformer shown in the Fig. 20, the voltage is reduced to one half and current increased twice ($n_p = 4$, $n_s = 2$). The transformer efficiency is given by:

$$\eta = \frac{U_s I_s}{U_p U_s} \cdot 100 \qquad [\%]$$

There are "copper" and "iron" energy losses in the transformer. The first ones are due to Joule's heat in copper winding wires, the second are due to the magnetization and demagnetization processes in the iron core.

Measuring instruments:
The construction of instruments for measuring electric currents, voltages and other electric quantities are beyond this textbook's scope. However, it is necessary to know that:
Voltmeters are characterised by very high intrinsic resistance. They must be connected in parallel to a "resistor" of which the voltage difference is to be measured. The measuring range of a voltmeter can be enlarged by "series resistors".
Ampermeters (ammeters) are characterised by very low intrinsic resistance. They must be connected in series into a circuit. The measuring range of an ammeter can be enlarged by "shunts" – small resistors added in parallel.

Problem example:
What is the kinetic energy of an electron moving in circular trajectory with radius $r = 10$ cm between the two poles of an electromagnet which produces a homogeneous magnetic flux density of 1 mT. The plane of the circular motion is perpendicular to the magnetic force lines. The mass of the electron is 9.11×10^{-31} kg.
Solution:
The kinetic energy of a moving body is given by the formula:

$$E_k = \frac{1}{2} m v^2$$

We have to calculate the electron velocity at first. The magnetic force of the magnetic field exerted on the electron must be equal to the centripetal force (the electron is on a circular path):

$$F = Bev\sin\Theta = Bev = m \cdot \frac{v^2}{r} \quad (\sin\Theta = 1)$$

After rewriting:

$$v = \frac{Ber}{m}$$

thus:

$$E_k = \frac{1}{2} m v^2 = \frac{B^2 e^2 r^2}{2m}$$

Result: After substitution and calculation we obtain 1.408×10^{-16} J. (This is only an approximate value. For correct calculation, relativistic correction of electron mass would be necessary since the electron moves with velocity of 1.75×10^4 km.s^{-1}.)

Problems to solve:

18. An "infinite" very thin wire is placed in vacuum, and carries an electric current of 10 A. What is the distance at which the value of the magnetic flux density B is equal to 1 T?

19. What is the reactance of a capacitor of capacity 1 μF in a circuit of alternating electric current of frequency 1 kHz?

20. What is the voltage induced in a single wire loop (one turn of a solenoid) of diameter 10 cm after switching on a homogeneous magnetic field (B = 0.1 T) in 1 ms. Assume that the B value increases linearly during this time. The wire loop is placed in vacuum.

21. What is the peak-to-peak value of the commonly used mains voltage ($U_{eff} = 230$ V).

9 OPTICS

9.1 Basic terms

Optics is the part of physics which deals with the propagation and wave properties of light. Light is defined as transverse oscillations of an electromagnetic field which propagate in discrete quanta of energy which can be considered particles (see also Fig. 30). These particles have zero rest mass, they are called **photons.**

The basic properties of light are given by its energy which is directly proportional to the frequency of its oscillations and inversely proportional to their wavelength. We can distinguish between three "types" of light. These are:

Visible light (abbreviation: VIS). Its wavelength λ ranges from 380 to 790 nm. This light can be detected by the retinal receptors of the human eye.

Ultraviolet light (UV). Its wavelength λ is shorter than 380 nm. Oscillations with wavelength shorter than about 10 nm are called X-rays; they are beyond the scope of classical optics, and will be discussed later.

Infrared light (IR). Its wavelength λ is longer than 790 nm. This radiation is also called heat radiation or thermal radiation, because it is radiated by hot bodies. Oscillations with wavelength longer than about 1 mm are called microwaves. They are the subject matter of theory of electromagnetism.

Note: We do not consider vector character of sme quantities in this chapter.

Optical medium (basic statements):

1. Any medium in which a light can propagate is called an **optical medium.** In general, we can distinguish between media which are **transparent** (we can see through them), **translucent** (not transparent but light can propagate through them, e.g. the so-called frosted glass) or **opaque** (light cannot propagate through them). The media in which optical properties are identical in all the directions, are called optically homogeneous and isotropic media.
2. In homogeneous media, light propagates in straight lines perpendicular to its wave fronts. These lines are called **light rays.**
3. The contact area between the two different optical media is called the optical **boundary (interface).**

Speed of light:

The speed (velocity) of light (in a vacuum) *c* is one of the most important physical constants:

$$c = 299\,792\,458\ m.s^{-1} \approx 3\times10^{8}\ m.s^{-1}$$

It is the highest speed which can be approached by a physical body with non-zero rest mass. The speed of light in vacuum was used for the modern **definition of the meter**:
1 meter is the distance which is travelled by light in vacuum during $\frac{1}{299792458}$ *s.*

Reflection and refraction of light:
On an optical boundary, light rays aiming from medium "1" to medium "2" can be reflected or refracted. Reflected rays do not penetrate into the medium "2"; the refracted light rays do that.

Reflection:
The reflection of light is described by the **law of reflection**:
The angle of reflection α' equals the angle of incidence α. The reflected ray travels in the plane of incidence.
The plane of incidence is the plane containing the incident beam, and it is perpendicular to the optical boundary. Both angles have to be measured away from the line perpendicular to the boundary.

Refraction:
The direction of a light ray changes when it passes through a boundary between two media. This property of optical media is characterised by the **index of refraction**:

$$n = \frac{c}{v}, \qquad \text{[dimensionless]}$$

where n is called the index of refraction of the respective medium, c is the speed of light in a vacuum, and v is the speed of light in respective medium. According to this formula, the index of refraction of a vacuum must be equal to 1.
If we have two optical media denoted as "1" and "2", the refraction of light is described by **Snell's law** (law of refraction – see Fig. 21.):

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1} = \frac{v_1}{v_2},$$

where α is the angle of incidence (measured in medium "1"), β is the angle of refraction (measured in medium "2"), n_1 and n_2 are the indices of refraction, v_1 and v_2 are the speeds of light in the respective media. Both angles have to be measured away from the line perpendicular to the boundary.

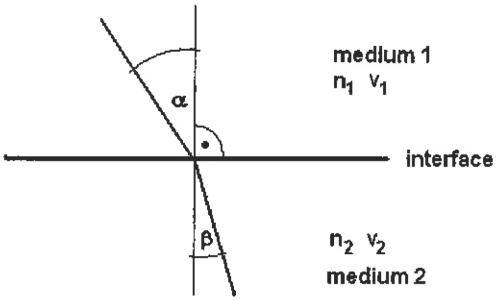

Fig. 21. Snell's law. See the description in the text.

For $n_1 < n_2$, the refraction is towards the line perpendicular to the boundary ($\alpha > \beta$). See Fig. 21.
For $n_1 > n_2$, the refraction is away from the line perpendicular to the boundary ($\alpha < \beta$).
Optical density:
We speak about a high optical density of an optical medium when its value of index of refraction is large. In the opposite case, we speak about low optical density.

Critical angle:
Consider a light beam which passes from the optical medium "1" to the medium "2", with $n_1 > n_2$. The critical angle is the angle of incidence for which the angle of refraction is 90°.

$$\frac{\sin\alpha}{\sin\beta} = \frac{\sin\alpha}{\sin 90^\circ} = \sin\alpha = \frac{n_2}{n_1}$$

Suppose the medium "2" is air for which $n \approx 1$. Thus:

$$\sin\alpha = \frac{1}{n_1} \quad \text{or} \quad n_1 = \frac{1}{\sin\alpha}$$

where α is the critical angle. The light travels similarly also in the opposite direction. It means that light rays coming from medium "2" to medium "1" with an angle of incidence almost equal to 90° are refracted at the critical angle.
It is obvious that critical angle measurement (based on the previous sentence) can be used for the determination of the index of refraction, i.e. in **refractometry**. A simplified scheme of an Abbe refractometer is shown in Fig. 22.

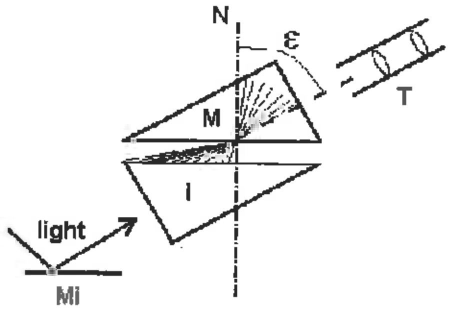

Fig. 22. Abbe refractometer. The narrow gap between the matted surface of the illumination prism (IP) and the polished surface of the measuring prism (MP) is filled by the liquid under study. The angle of incidence on the surface of MP takes values from 0 to 90°. The critical angle ε is the maximal angle of refraction. When the optical axis of the observation system ("telescope", T) is in the same direction as the rays refracted at the critical angle, it is possible to see a boundary between light and dark areas in the "telescope". M is a mirror, and N is the normal (perpendicular line) to the gap between prisms.

Total internal reflection:
The angle of incidence can be also greater than the critical angle. In this case, all the light rays are only reflected, and there is no refraction. This phenomenon is often used in optical instruments (totally reflecting prisms, optical fibres etc.).

9.2 Optical imaging by lenses and mirrors

Common principles of optical imaging:
Refracting optical boundaries can be used for image formation. A **real image** can be projected onto a screen; it is formed only by convergent light rays. A **virtual image** cannot be projected; it is formed by divergent rays.
Remember the following terms:
Principal axis – it is the axis of a centred system of optical boundaries (e.g. lenses, mirrors)
Principal focus – it is the point where rays parallel to the principal axis intersect (or seem to emerge from) after being refracted by a lens or reflected by the curved mirror.
The **focal distance** is the distance of the principal focus from the centre of the lens or the mirror.

The image formed by a **planar (flat) mirror** is always virtual ("can be seen only in the mirror"), erected ("standing"), of the same dimensions as the object, and symmetrical with the object (mirror is the plane of symmetry).

Note: Most of the following equations, formulas or principles are true (valid) only for thin lenses and for rays close to the principal axis (paraxial rays). The lens or mirror surfaces are a part of a spherical surface.

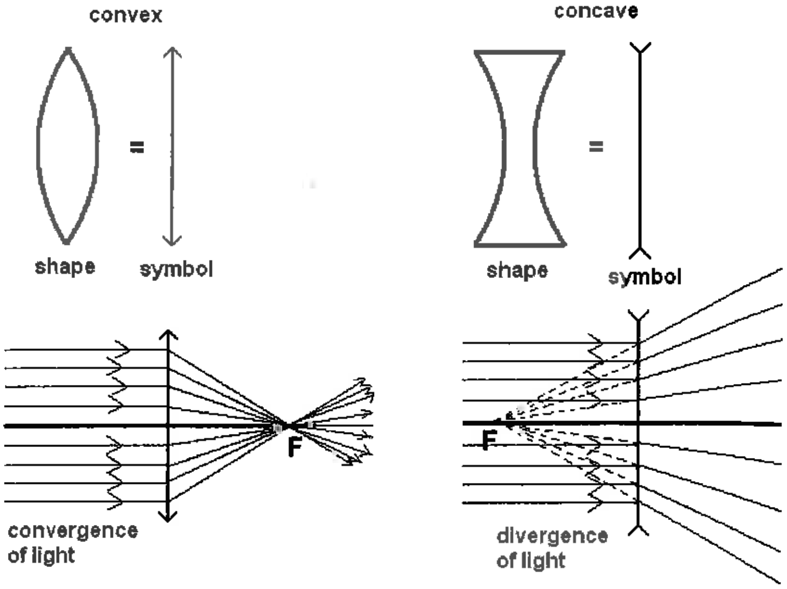

Fig. 23. Converging and diverging lens – shapes, symbols and passage of light rays. In the converging lens, the rays coming in parallel to the principal axis are refracted *into* the focal point. In the diverging lens, such rays seem to come *from* the focal point.

Lenses:

In principle, lenses can be converging (with convex shape) and diverging (with concave shape) – see Fig. 23. The relationship between the geometry of the lens (radii of their curvature), their index of refraction, and their focal distance is given by the **lens-maker's equation**:

$$\frac{1}{f}=\left(\frac{n_2}{n_1}-1\right)\cdot\left(\frac{1}{r_1}+\frac{1}{r_2}\right),$$

where f is the focal distance [m] of the lens, n_2 is the index of refraction of the lens, n_1 is the index of refraction of the optical medium surrounding the lens, and r_1 and r_2 are the radii of curvature [m] of the lens.

The following internationally accepted **sign convention** must be used: *The radii of curvature are positive (negative) when the respective lens surfaces are convex (concave).*

Dioptric power:

This quantity expresses the "strength of the lens". It is defined as the reciprocal value of the focal distance:

$$\phi=\frac{1}{f} \qquad [\mathrm{m}^{-1} = \mathrm{dpt} = \mathrm{D}\ (\text{dioptre})]$$

There is also an internationally accepted **sign convention**: *The focal distance f and the dioptric power ϕ are positive in converging lenses. In diverging lenses, f and ϕ are negative.*

The total dioptric power of a series of lenses having the same principal axis is given by the sum of dioptric powers of individual thin and close lenses:

$$\frac{1}{f} = \phi = \frac{1}{f_1} + \frac{1}{f_2} + \ldots = \phi_1 + \phi_2 + \ldots$$

To understand the formation of an image by a spherical lens (shown in Fig. 24), it is necessary to know that the rays parallel to the principal axis are refracted into the back focus (in a converging lens), or that they seem to be emitted from the front focus (in a diverging lens). The direction of rays passing through the centre of the lens remains unchanged.

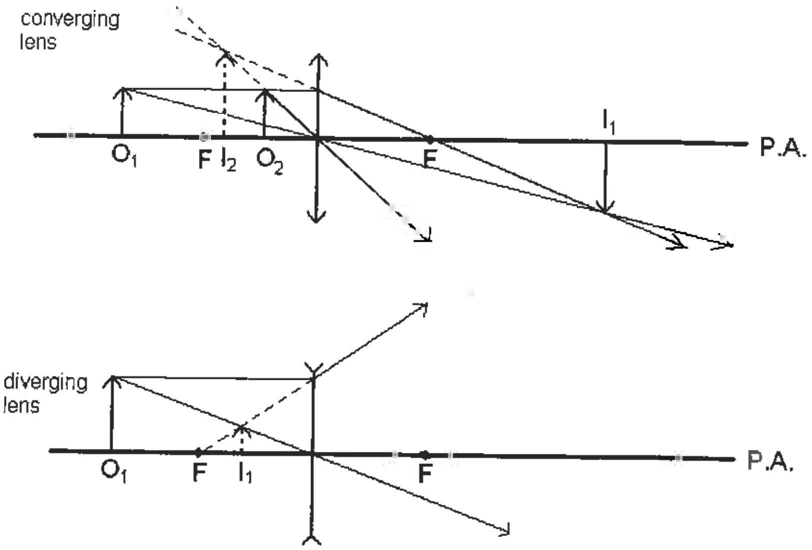

Fig. 24. Schematic drawing of image formation by converging and diverging lenses. I – images, F – focal points, O – objects, P.A. – principal axis. Only the image I_1 is real.

Some basic calculations can be done by means of the **lens equation:**

$$\frac{1}{a} + \frac{1}{a'} = \frac{1}{f},$$

where a is the object distance [m] from the centre of the lens, and a' is the image distance from the centre of the lens (object distance or image distance).

The following sign convention must be applied when using the lens equation for any calculations: *a is positive in front of the lens, negative behind the lens; a' is negative in front of the lens (the image is virtual and erected), and positive behind the lens (the image is real and inverted).*

It is possible to derive some useful formulas which are often used for various calculations (see the Problems):

$$M = \frac{y'}{y} = -\frac{a'}{a} = -\frac{a'-f}{f} = -\frac{f}{a-f},$$

where M is the dimensionless quantity called linear (also transverse or lateral) magnification.
If $M > 0$ (a positive number) the image is erected.
If $M < 0$ (a negative number) the image is inverted.
If $M > |1|$ (M is greater than the absolute value of number 1) the image is magnified (greater).
If $M < |1|$ (M is smaller then the absolute value of number 1) the image is diminished (smaller).
y is the height of the object [m].
y' is the height of the image [m].

Real lenses have some **aberrations.** We mention two important aberrations:
a) **Chromatic aberration** is caused by different values of refraction indices for light rays of different wavelength. i.e. different colours of light ($n_{red} < n_{yellow} < n_{violet}$). It means that the dioptric power of a lens depends on the colour of light. The rays of different colour are refracted to different foci.
b) **Spherical aberration** is due to imperfectness of the spherical surfaces of lenses. The focus is not a single point; the rays intersect in a certain volume after being refracted. This aberration can be often observed in thick lenses.

Mirrors:
We can distinguish between converging (i.e. concave) mirrors, and diverging (i.e. convex) mirrors.
The image equation for mirrors is very similar to that for lenses:

$$\frac{1}{a}+\frac{1}{a'}=\frac{1}{f}=\frac{2}{r}$$

where *a* is the object distance (distance of the object from the centre of the mirror, i.e. from the point at which the principal axis of the mirror intersects the mirror surface) [m], *a'* is the image distance (distance of the image from the centre of the mirror) [m], *r* is the radius of mirror curvature (note that the focus of a mirror lies in the middle between the centre of the mirror and the centre of the curvature, $r = 2f$).
The rays parallel to the principal axis are reflected into the focus (in concave mirrors), or in such a way that they seem to be emitted from the focus (in convex mirrors). The rays passing through the centre of curvature of the mirror fall perpendicularly onto the mirror surface, and they retrace their path after reflection.

Sign convention:
a, a', r, f are positive in front of the mirror, and negative behind the mirror,
$a' > 0$ – real image
$a' < 0$ – virtual image
Useful formulas derived for mirrors are similar to that of lenses:

$$M=\frac{y'}{y}=-\frac{a'}{a}=-\frac{a'-f}{f}=-\frac{f}{a-f},$$

where *M* is linear (also transverse or lateral) magnification [dimensionless]. If $M > 0$ the image is erect. If $M < 0$ the image is inverted. If $M > |1|$ the image is magnified (greater), and if $M < |1|$ the image is diminished (smaller). *y* is the height of the object [m], and *y'* is the height of the image [m].

9.3 The human eye and simple optical instruments

Human eye:
In the following paragraphs, we will shortly explain some of the important terms of anatomy, physiology and biophysics of the eye. A simplified anatomy of the human eye is shown in **Fig.** 25.
The **eye ball** is a sensory organ serving for our **vision.** It is almost a sphere covered with an outer white layer called the **sclera.** It is elastic and relatively rigid. In the front part of the eye, the sclera becomes transparent, and it is called the **cornea.**

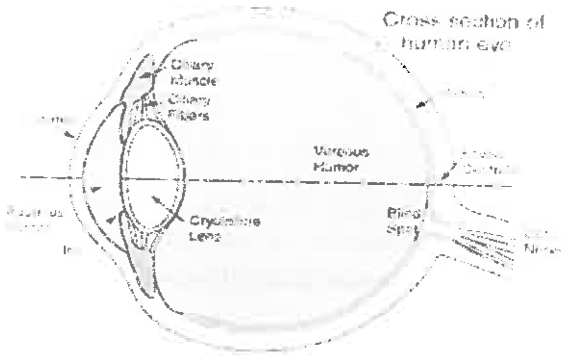

Fig. 25. Anatomy of human eye

The optical media of the eye are the following: cornea, **aqueous humour** (a liquid between the cornea and the lens), **crystalline lens**, and **vitreous humour** (a gel-like substance which fills the eye ball). After passing through these optical media, the light rays fall on the innermost tissue layer of the eye – the **retina** – and form a **real inverted image** there.

Accommodation is the ability of the eye to change its dioptric power. It is ensured by a change of lens curvature due to the activity of the **ciliary muscles** (see Figs. 25 and 26). The **near point** is the minimum distance at which an object can be seen sharply, i.e. at which a sharp image is formed on the retina. Similarly, the **far point** is the maximum distance of an object allowing sharp vision. In a normally seeing **eye**, the far point lies at infinity.

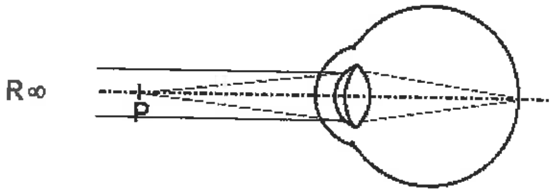

Fig. 26. Accommodation of the eye. To see sharply the near point P (*lat. punctum proximum*), the curvature of the lens is maximal. R – far point (*lat. punctum remotum*).

A frequently used term (see e.g. the paragraphs dealing with the microscope) is the **distance of most distinct vision** (also "**convention visual distance**") which value is 25 cm.

The two most important optical aberrations of an ametropic (i.e. not normally seeing) human eye are called myopia (nearsightedness) and hyperopia (farsightedness) – see Fig. 27. An eye capable of normal vision is called emmetropic. In the case of myopia, the dioptric power of the cornea and/or crystalline lens is higher than the normal value, and the image is formed in front of the retina. This aberration can be corrected by a diverging lens. In the case of hyperopia, the dioptric power of the cornea and/or crystalline lens is lower than the normal value, and the image would be formed behind the retina. This aberration can be corrected by a converging lens.

The **resolution threshold** (the ability to distinguish two points at a small distance from each other) of the human eye is given by a viewing angle of 1' (one minute of arc).

The **iris** regulates the amount of light entering the eyeball.

In the retina, there are photosensitive cells called **cones** (responsible for colour vision) and **rods** (ensuring vision in darkness). The cones are concentrated in the central part of the retina, which is called the **yellow spot**. The retina has specific cones for red, green and blue colours. The excitation of the photosensitive cell is due to the photochemical disintegration of substances called **visual purples – rhodopsin** (in rods) or **iodopsin** (in cones).

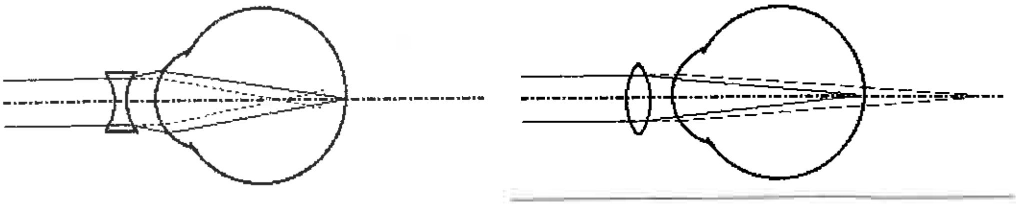

Fig. 27. Ametropic eyes and respective corrections. Left image: Myopia corrected by a diverging lens. Right image: Hyperopia corrected by a converging lens.

Optical instruments:

Now we will briefly explain the principles of two very important optical instruments – the magnifier and the microscope.

A **magnifier** or **magnifying glass** is any converging lens whose focal distance is smaller or equal to 25 cm, i.e. dioptric power equal or greater than 4 dpt.

The image formed by the magnifier is virtual, magnified and erect. See Fig. 24 – formation of the image I_2 in the converging lens.

Suppose that the observed object is placed within the focal distance of the magnifier. Then:

$$\gamma = \frac{d}{a},$$

where γ (gamma) is the angular magnification ($= \frac{\tau'}{\tau}$), τ' (tau-prime) is the virtual angle of vision, τ (tau) is the real angle of vision, *d* (25 cm) is the distance of the most distinct vision, and *a* is the object distance ($\approx f$).

Microscope:

Apart of the mechanical movable and supporting parts, the microscope is formed by an objective and an eyepiece. The simplest **objective** is a converging lens that forms a real, magnified and inverted image. The **eyepiece** is a converging lens used as a magnifying glass for observation of the image formed by the objective. Therefore, the resulting image is magnified, virtual and inverted. See Fig. 28.

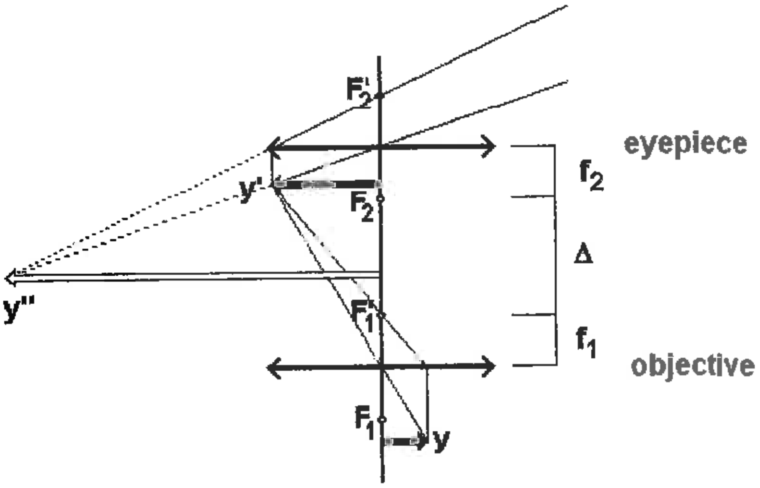

Fig. 28. F – focal points, *f* – focal distances, y - object, y' – real image of the object formed by the objective, y'' – virtual image seen in the eyepiece, Δ – optical interval of the microscope.

The total magnification of an optical microscope (in practice max. about 2000-times) is given by the product of the objective and eyepiece magnification:

$$M = M_{obj} M_{ep},$$

where M_{obj} is the magnification of the objective, and M_{ep} is the magnification of the eyepiece. The angular magnification of the microscope is given by the following important formula:

$$\gamma = \frac{\Delta d}{f_{obj} f_{ep}},$$

where Δ is the **optical interval** of the microscope (the distance between back focus of the objective and front focus of the eyepiece – see Fig. 28), d is the distance of the most distinct vision (25 cm), f_{obj} and f_{ep} are the focal distances of the objective and eyepiece.

9.4 Wave properties of light

Light can be described in terms of wave motion. The light waves are harmonic oscillations of an electromagnetic field – the vectors of magnetic flux density and intensity of electric field oscillate perpendicularly to the direction of light propagation.

Let us start from the **speed (velocity) of light**:

$$v = \frac{\lambda}{T} = \lambda f = \frac{c}{n},$$

where T is the period of oscillations [s], f is their frequency [s^{-1}], and λ is their wavelength [m]; n is the index of refraction, and c is the speed of light in vacuum. Remember that the previously mentioned index of refraction depends strongly on the frequency or wavelength of light. The index of refraction increases with decreasing wavelength. Remember the relation mentioned in the paragraph about chromatic aberration: $n_{red} < n_{yellow} < n_{violet}$.

Therefore, the most refracted light is the violet light. According to the last equation we can write:

$$f = \frac{c}{\lambda_0} = \frac{v}{\lambda}$$

then:

$$\frac{c}{v} = n = \frac{\lambda_0}{\lambda}$$

hence:

$$\lambda = \frac{\lambda_0}{n}.$$

where f is the frequency of the light, c is the speed of light in a vacuum, v is the speed of light in given medium, λ is the wavelength of light in the given medium, and λ_0 is the wavelength of light in vacuum.

Interference of light:

Interference of light and related phenomena can be observed with beams of coherent light rays, i.e. rays of the same frequency and the same phase shift at a given distance from the source of light. Interference is a result of superposition (addition, summation) of oscillating vectors characterising the electromagnetic field.

We will briefly discuss the interference of light on thin layers of transparent substances like air or water (i.e. the interference of two coherent light rays, the first of which is reflected from the surface of a thin layer, and the second one is reflected from the "bottom" of this layer of thickness *d*). See also Fig 29.

Premises for solving the problem:

a) **Optical path length**

$$l = ns$$

where *s* is a geometric path (distance). The optical path length is the distance which would be travelled by light in a vacuum (air) during the same time as the distance *s* in given medium. In Fig. 29, the optical path length of the depicted light rays differs by $2n_{water}d' - n_{glass}d$.

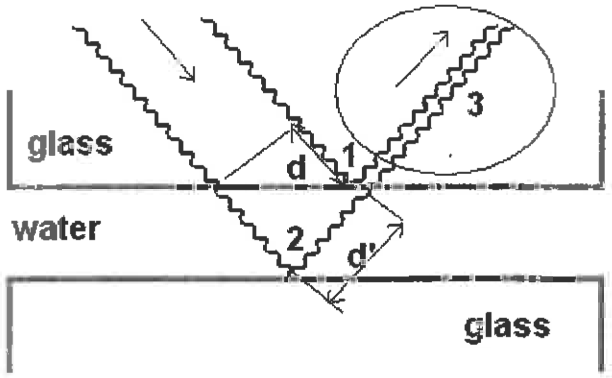

Fig. 29. To the interference of light on a thin layer. 1, 2 – reflecting rays, 3 – interfering rays. Light refraction is neglected.

b) Light oscillations change their phase to the opposite value when reflecting on an optically denser medium (it is an equivalent of path length change by λ/2).

The phase of light oscillations remains unchanged when they reflect on an optically rarer (less dense) medium.

c) **Interference maximum** (maximum intensity) of the reflected beams occurs when the following condition is fulfilled:

$$2nd + \frac{\lambda}{2} = 2k \cdot \frac{\lambda}{2} \quad \Rightarrow \quad 2nd = (2k-1) \cdot \frac{\lambda}{2},$$

where $k = 1, 2, 3\ldots$ are whole numbers (integers).

The above condition expresses the fact that the optical path length in a thin layer of a transparent medium (*2n.d*) must be equal to an odd number of λ/2.

d) **Interference minimum** (minimum intensity) of the reflected beams occurs when:

$$2nd + \frac{\lambda}{2} = (2k+1) \cdot \frac{\lambda}{2} \quad \Rightarrow \quad 2nd = 2k \cdot \frac{\lambda}{2},$$

where $k = 1, 2, 3\ldots$ are whole numbers (integers).

This condition expresses the fact that the optical path length in a thin layer of a transparent medium (*2nd*) must be equal to the even number of λ/2.

When observing a thin layer of non-constant thickness, the interference maxima and minima produced by monochromatic light can be seen as bright and dark areas, respectively. The interference of a white (polychromatic) light produces coloured patterns (e.g. stripes).

Diffraction of light:
Light diffraction is a change of light propagation direction which occurs when light is incident on particles, holes, slits etc. the dimensions of which are comparable with the wavelength of light. It often demonstrates itself as the scattering of light. Light rays diffracted at openings, slits, double slits or diffraction gratings can interfere with each other. Therefore, we can observe also diffraction-conditioned interference phenomena.

Polarised light:
As already explained, visible light consists of electromagnetic waves which are characterised by vectors of ***E*** (electric field intensity) and ***B*** (magnetic flux density). These vectors oscillate perpendicularly to the direction of light propagation (see Fig. 30). If the light beam consists of more rays, their B and E vectors can either oscillate in many planes or only in a single plane (two perpendicular planes when considering both B and E). Such a plane is called the **polarisation plane.** In the first case we speak about non-polarised light, in the second case about **polarised light**.
Light waves can be polarised by several different means:

- by **polarisation filters (polaroids,** e.g. in sun glasses). Such a filter can be passed through only by light the polarisation plane of which has an orientation allowing the passage.
- by **reflection** (the light reflected from glass, mirrors, polished surfaces etc. is partly polarised – this causes reflected light is reduced in intensity by polarised sun glasses.
- by **double refraction (birefringence,** encountered in anisotropic crystals, e.g. the Nicol prism). In this phenomenon, the original ray is split into two parallel rays polarised perpendicularly.

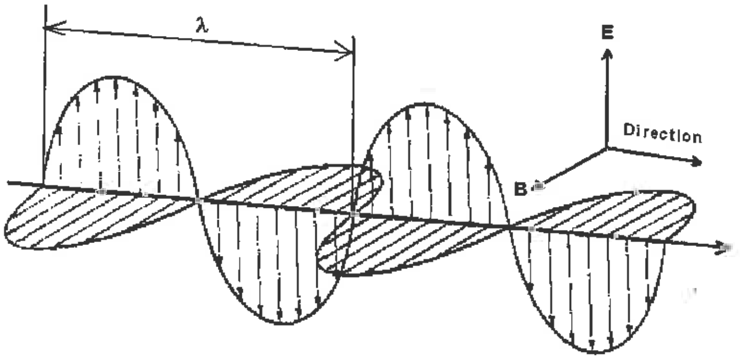

Fig. 30. Electromagnetic character of light. B – magnetic flux density, E – intensity of electric field, λ = wavelength.

Optically active substances (in a crystal form or in a solution) are able to rotate the plane of polarised light. Many organic substances of great biological importance are optically active. It is possible, for example, to measure the rotation angle of polarised light which depends on the concentration of the substance through which the polarised light passed. The concentration of sugars is often measured in this way. The measurement is done by means of instruments called **polarimeters.**

9.5 An introduction to photometry

A relatively frequent task is to measure the quantity of light emitted by a body or incident on an illuminated surface. Therefore we will briefly mention some of the most important photometric quantities used in the SI system. Note: We always assume that the source of light radiates equally in all directions.
The **luminous intensity** I is the measure of light produced by a source of unit area in unit time. Its unit, **candela** [cd] belongs among the fundamental SI-units. The candela can be defined by the following statement:

1 cd is one sixtieth of the luminous intensity of a "black body" source, 1 cm² in cross-section area, at the temperature of the platinum melting point (1755 °C), under normal atmospheric pressure and viewed at right-angles to the area.

A new definition since 1979: *The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 × 10^{12} Hz and that has a radiant intensity in that direction of $^1\!/_{683}$ watt per steradian*

Note: A black body does not reflect light, it can only emit light.

Luminous flux Φ is the measure of the amount of light emitted into a defined *solid* angle Ω. Its unit is called **lumen** [lm]:

1 lm is the luminous flux produced by a point source of luminous intensity 1 cd into a solid angle of 1 sr (steradian).

Suppose that a 1-cd point source is at the centre of a hollow sphere with radius $r = 1$ m (called "unit sphere"). A luminous flux of 1 lm will pass through an area of 1 m² on the sphere surface.

$$\Delta\Phi = I\Delta\Omega \quad \Leftrightarrow \quad I = \frac{\Delta\Phi}{\Delta\Omega} \qquad [\mathrm{lm}] \quad [\mathrm{cd}],$$

where $\Delta\Omega$ is the solid angle in steradians [sr].

Note: You can imagine the solid angle as a cone without a base. You know that the *radians* – units of plane angle – are derived from the *unit circle* (r = 1 m) which circumference is $2\pi r = 2\pi$. The *round* plane angle (perigon) is 2π radians; one radian intercepts unit length (1 m) on a unit circle. The *steradians* – units of solid angle – are derived from the *unit sphere* (r = 1 m) with surface area $4\pi r^2 = 4\pi$. The *round* solid angle is 4π steradians; one steradian intercepts unit area (1 m²) on a unit sphere.

The **illumination** (illuminance) E [lm.m^{-2} = lx – lux] expresses the amount of light incident onto a surface. Definition:

Luminous flux of 1 lm per 1 m² produces an illumination of 1 lux.

Thus

$$E = \frac{\Delta\Phi}{\Delta A} \qquad [\mathrm{lx}],$$

where we can substitute for the illuminated area ΔA and the luminous flux $\Delta\Phi$:

$$\Delta A = r^2\Delta\Omega \qquad \text{and} \qquad \Delta\Phi = \Delta\Omega I .$$

Hence:

$$E = \frac{\Delta\Omega I}{r^2\Delta\Omega} = \frac{I}{r^2} \quad \left(= \frac{I\cos\alpha}{r^2} \right) \qquad [\mathrm{lx}],$$

where α is the angle of incidence (measured away from the line perpendicular to the illuminated surface), A is the area of illuminated surface, and r is the distance of illuminated surface from the source of light. The last formula can also be read in the following way: *The illumination of a surface varies inversely as the square of its distance from a point source.*

Problem example:

*In a thin converging lens, an image is formed 10 cm **in front** of a lens at a linear magnification of 4. What are the focal distance and dioptric power of this lens?*

Solution:

We have to use these two formulas:

$$M = -\frac{a'}{a} \quad \Rightarrow \quad a = -\frac{a'}{M} \qquad and \qquad \frac{1}{f} = \frac{1}{a'} + \frac{1}{a}$$

$a' = -10\,cm = -0.1\,m;\ M = 4$

Thus, after substitution of numerical values: $a = 0.025\,m$

Substitute in the second formula:

$$\frac{1}{f} = -\frac{1}{0.1} + \frac{1}{0.025} = -10 + 40 = 30\,dpt \quad \Rightarrow \quad f' = 0.0333\,m$$

Result: The focal distance of the lens is 3.33 cm while its dioptric power equals 30 dpt.

Problems to solve:

22. A thin symmetric lens ($r_1 = r_2 = 50\,cm$) of dioptric power 2 dpt is placed in air. What is the value of the lens refractive index?

23. What is the focal distance of a mirror of linear magnification 10, for an image at a distance of -90 cm?

24. What is the angular magnification of a microscope of optical interval 15 cm, objective focal distance 2 mm and eyepiece focal distance 10 mm?

25. What is the wavelength of infrared light which travels in a vacuum with frequency of 2000 GHz?

26. What is the luminous intensity of a light source (emitting light equally in all directions) which illuminates by 400 lx a surface oriented at right angles? The surface is placed at a distance of 3 m from the source.

10 THEORY OF RELATIVITY

In this chapter we will only mention some of the most frequently used principles and formulas without proof. The theory of relativity is based on the two following principles:

1. **The principle of relativity: The laws of physics are the same in all inertial reference frames** (co-ordinate systems). (It means that it is not possible to determine, which inertial reference frame is at rest or in motion. Reference frame is inertial if it is at rest or in uniform rectilinear motion).
2. **The principle of the constancy of the speed of light: The speed of light in free space has the same value c in all inertial reference frames.**

Dilation of time:

("in reference frames in relative motion, time passes more slowly")

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Δt – the time interval measured in a reference frame in relative motion ("a clock in a rocket"),
Δt_0 – the time interval measured in a reference frame at relative rest ("a clock on the Earth"),
v – relative velocity of the moving reference frame,
c – speed of light in vacuum.

Contraction of length:

("The length of a body measured by an observer who is at rest with respect to the object being measured is called the rest length. All observers in motion relative to the first observer measure a shorter length but only for dimensions along the direction of motion.")

$$l = l_0 \cdot \sqrt{1 - \frac{v^2}{c^2}},$$

l – length of a body measured by the observer in motion with respect to the body,
l_0 – length of a body measured by the observer at rest with respect to the body,
v – relative velocity of the observer with respect to the observed body,
c – speed of light in vacuum.

Relativistic addition of velocities (see Fig. 31):

When a body B moves with a velocity *u'* in the direction of the x-axis with respect to the reference frame S', and this frame simultaneously moves with respect to the reference frame S at velocity *v* in the same direction, the body B then moves with respect to S at velocity *u*:

$$u = \frac{u' + v}{1 + \frac{v.u'}{c^2}}$$

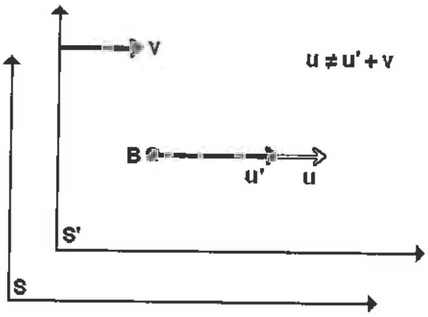

Fig. 31. Relativistic addition of velocities. See the text for description.

Relativistic mass:

("the mass of a body increases with its velocity")

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where m_0 is the rest mass of the body, and m is its mass at the speed v.

Energy and mass equivalence principle (Einstein energy equation):

Under certain circumstances, energy can be transformed into mass or mass can be transformed into energy. The following formulas belong to the general theory of relativity (contrary to the previous formulas which are involved in the special theory of relativity).

$$\Delta E = \Delta m c^2$$

$$E = m.c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ (in motion)}$$

$$E_0 = m_0 c^2 \text{ (at rest)}$$

All the relativistic phenomena are well manifested when the velocities are comparable with the speed of light. They are scarcely observed in everyday life. The theory of relativity is of special importance in physic of elementary particles and in cosmology.

Problems to solve:
27. Free neutrons (i.e. neutrons outside the atom nucleus) are elementary particles with a lifetime of about 1010 s. After this time we have only one half of the original number of free neutrons. Let the neutrons move at speed of 200 000 km/s. What is the new lifetime of the neutrons when measured by an observer at rest? (Solve it as a case of time dilation)
28. What is the relativistic mass of the fast neutrons described in the previous problem? Assume that the mass of a neutron at rest equals 1.

11 PRINCIPLES OF QUANTUM, ATOMIC AND NUCLEAR PHYSICS

11.1 Introduction

The principles and applications of modern physics, which encompasses a lot of quantum, atomic and nuclear physics, are extremely important for biomedical sciences. Almost all imaging methods used in medicine are based on this part of physics.
Before we start to describe the processes occurring on the atomic and subatomic level of matter, it is necessary to define a "new" unit of energy, which is very often used when speaking about individual particles and their small assemblies – molecules. The main SI unit of energy – the joule – is too big for that purpose. Therefore, the small amounts of energy carried by individual elementary particles, atoms, chemical bonds etc. are measured in units called electron volts:

$$1\ \text{electron volt}\ [eV] = 1.602\times10^{-19}\ J$$

1 electron volt is the energy obtained by one electron when accelerated in an electrostatic field by 1 V potential difference. The potential energy of a charged particle $E = W = QU$ (in our case eU) changes into its kinetic energy during the acceleration.

Basic properties of atoms:
1. 1 mole is such an amount of a substance which contains such number of particles (atoms, molecules, ions ...) as present in a sample of the nuclide carbon-12 which mass is equal to 0.012 kg.
The number of particles in 1 mole of any substance is called the **Avogadro's constant**:

$$N_A = 6.022\times10^{23}\ mol^{-1}.$$

2. Atomic mass constant (or unit) is 1/12 of the mass of carbon-12 nuclide atom:

$$m_u = 1.66\times10^{-27}\ kg$$

3. Molar mass (M_m) is the mass of 1 mole of a substance (it is equal to the sum of atomic masses of the atoms forming one molecule expressed in grams)
4. Molar volume is the volume of 1 mole of a substance:

$$V_m = \frac{M_m}{\rho}$$

where ρ (ro) is density of the substance.

Each atom consists of a nucleus and electron shells.

The **internal energy** of atoms is quantized (it means that only discrete values of energy are possible or achievable).
Excitation is a process in which an atom absorbs energy of any kind. The **deexcitation** of atoms is accompanied by liberation of energy in the form of a **photon** which is a defined quantum of energy of electromagnetic oscillations. We can also speak about the ground state and excited state of an atom.
The **energy of a photon** can be calculated using Planck's formula:

$$E = h\nu = hf,$$

where h is the Planck's constant (6.63×10^{-34} Js), and ν (nu) or f the frequency ($f = c/\lambda$).

Wave properties of particles:
It is not possible to regard atoms or subatomic particles (e.g. electrons, protons, neutrons) only as small rigid balls because they also possess some wave properties. Their wave motion (oscillation) manifests itself as **de Broglie (matter) waves**, with wavelength given by the following formula:

$$\lambda = \frac{h}{p} = \frac{h}{mv},$$

where λ is the wavelength of the de Broglie waves [m], h is the Planck's constant (6.63×10^{-34} Js), and p is the momentum of the particle [$\mathrm{kgms^{-1}}$].
Crucial evidence for the wave properties of the above mentioned particles are the diffraction and interference phenomena which can be observed under special conditions (e.g. after passage of electrons through a crystal lattice).

Wave function:
The wave function is a complex mathematical function (a solution of the Schrödinger equation) determining the possible states of electrons located in an electromagnetic field. There is no need to understand this function for medical students but it is important to know that the absolute squared value of this function expresses the probability that a particle (e.g. an electron) exists in a certain part of space (say, in the close vicinity of the atomic nucleus).

11.2 Properties of electron shells

Quantum mechanics model of the hydrogen atom:
The wave function is obtained as a solution of the Schrödinger equation for an electron occurring in the electrostatic field of a positively charged atomic nucleus only for certain values of parameters called **quantum numbers**. These numbers are four:

1. **Principal quantum number** n. It can take values 1, 2, 3, 4...
The respective electron shells are marked by capital letters K ($n = 1$), L ($n = 2$), M ($n = 3$), N ($n = 4$)...
The principal quantum number determines mainly the energy of electrons.
For the simplest atom hydrogen ^1_1H we can write:

$$E_n = -\frac{E_1}{n^2},$$

where E_1 is the energy of the ground state of the electron ($n = 1$). Note that the value of energy is negative because some work must be done to liberate the electron from the shell.
Electrons occurring in the innermost shells have the highest negative energy. There we also find the greatest energy differences between shells.

2. **Orbital momentum quantum number** l. It can take value 0, 1, 2, 3 ... $n - 1$ for each principal quantum number n.
The respective orbitals, i.e. the defined parts of space in which the electrons can be found with highest probability, are marked by letters s ($l = 0$), p ($l = 1$), d ($l = 2$), f ($l = 3$)...
These numbers determine the angular momentum of the electron, i.e. the shape of the orbital (see textbooks of chemistry for detailed explanation).

3. **Magnetic quantum number** m. This can take values 0, ±1, ±2, and ±3 ... $\pm l$ for each orbital quantum number l.
These numbers determine the allowed spatial orientations of electron orbitals in an external magnetic field.

4. **Spin quantum number** s. It can take only two values: $\pm\frac{1}{2}$.
This number quantizes the intrinsic angular momentum of an electron, which can be imagined as being the rotation of an electron (say, clockwise and anti-clockwise).

Pauli's exclusion principle states:
No two electrons in an atom can exist with the same set of four quantum number values.
In atoms with a higher number of electrons, the energy of the electron depends also partly on the quantum numbers l, m, and s.

Spectral analysis:
Various physical bodies can absorb or emit ultraviolet, visible or infrared light. This phenomenon can be explained on the basis of excitation and deexcitation processes. Light is emitted mainly from hot bodies in which the atoms (or molecules) become repeatedly excited and then returning to the ground energetic state by deexcitation. Light absorption can be considered an important excitation process.
The dependence of intensity of emitted or absorbed light on light wavelength is called a **spectral curve**. To obtain such a curve, it is necessary to disperse (separate) polychromatic light into its components which differ in their wavelength. It can be done e.g. by prisms because the index of refraction depends on wavelength so that light of different wavelength is refracted through various angles. Another possibility is the use of diffraction gratings.
The instruments used to study absorbed and emitted light are called spectrographs (these serve only for visualization of absorption or emission spectra on a screen) or **spectrophotometers** (these serve for objective measurement of light absorption or emission).
The **emission spectrum** is obtained after passage of the light beam emitted from the substance under study through the prism or grating. The **absorption spectrum** can be seen and measured after passage of polychromatic light through a solution of the substance under study followed by spectral analysis of the transmitted light.

In gases formed by atoms or very simple molecules, only light of certain (discrete) values of wavelength is absorbed or emitted. In such a case, we can see only bright lines on a black background (in emission spectra) or dark lines on a background of spectral colours (in absorption spectra) – we speak about **line spectra** in both cases. Many molecules are so sensitive to high temperatures that they cannot emit light due to heating. Complex molecules dissolved in solution absorb light in a relatively wide interval of wavelength values. Thus, we can speak about **band spectra**. The **continuous spectra** of emitted light are found in hot solid bodies.

The existence of line spectra can be simply explained taking into account the discrete values of electron energies in atomic electron shells (they are given by the principal quantum numbers, $n = 1, 2, 3 ...$). Therefore, the changes of electron energy and energy values of emitted photons can also have only discrete values. The band or continuous character of spectra of some substances is the result of mutual interactions between many electron orbitals. The electron energy is then not discrete, it has many, or an almost infinite number of possible values.

Origin of X-rays:

X-rays are beams of electromagnetic radiation which originate in atomic electron shells. They are formed by photons of very high energy (substantially higher than the energy of ultraviolet light photons). In practice, they are produced in X-ray tubes – evacuated glass tubes involving two electrodes – the anode and the cathode. See Fig. 32.

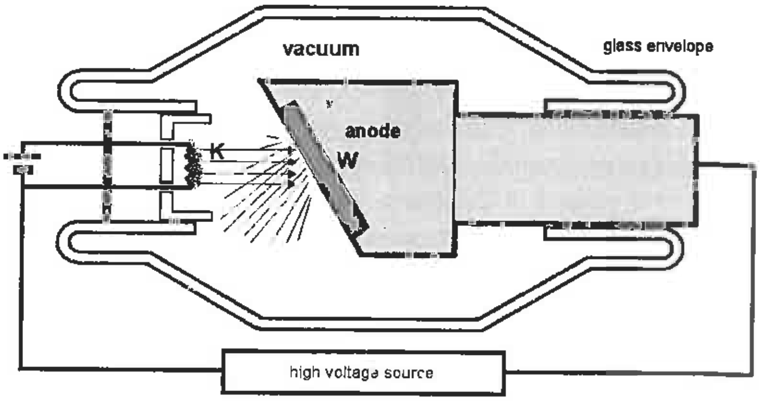

Fig. 32. The X-ray tube. K – cathode (hot filament, sometimes the whole cathode is called filament), W – tungsten target

Electrons escape from the hot cathode surface (thermoemission of electrons), and then they are rapidly accelerated in the space between the hot cathode and the (cold) anode by very high electric voltage (e.g. 150,000 volts. The energy of electrons striking the anode is given by the term $½m.v^2$ which is equal to their original potential energy Ue. Subsequently, these electrons are suddenly decelerated in a tungsten (W) target which is a part of the anode. It means that their kinetic energy is transformed into X-ray and heat energy. However, on average, only a small part (less than 1 %) of this energy is transformed into high energy X-ray photons. More than 99 % of their energy is transformed into heat. In the rare case of total electron energy transformation into a single X-ray photon, the energy of the photon is:

$$E = hf_{max} = \frac{1}{2}mv^2 = Ue = \frac{hc}{\lambda_{min}} \qquad (f = c/\lambda)$$

where hf_{max} is the maximum energy of the liberated photon. Such photons possess maximum possible frequency f_{max} or the shortest possible wavelength λ_{min}.

The energy spectrum of these X-rays is continuous – see Fig. 33

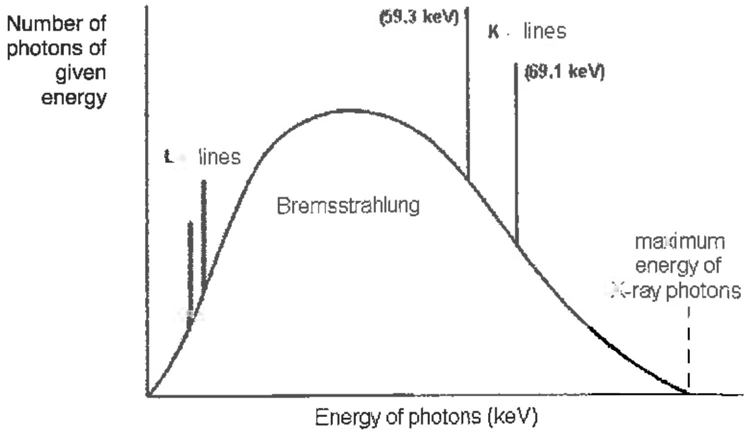

Fig. 33. Schematic representation of a spectrum of X-rays. See the text for more detailed description.

The X-rays produced in this way are called "**Bremsstrahlung**" (i.e. deceleration radiation) which is a German word used in honour of the German physicist W.C. Roentgen, the discoverer of X-rays.
X-ray photons can be also produced by jumps of electrons from outer to the innermost shells of some heavy atoms. There are vacancies (unoccupied sites) formed by impact of accelerated electrons, which must be filled immediately by some other electron. The spectrum of X-rays produced in this way consists of lines representing discrete energies of photons. This is the **characteristic radiation** – see also Fig. 33.)

Photoelectric effect:
We can observe this with electromagnetic radiation the energy of which is high enough to liberate electrons from matter.
We already know that electromagnetic radiation propagates in the form of small quanta of energy, which are called photons. The energy of one photon is given by Planck's formula:

$$E = hf,$$

where h is the Planck's constant and f is the frequency. The photoelectric effect is an energy transformation which is described by Einstein formula:

$$h.f = W_b + \frac{1}{2}mv^2 \ ,$$

where hf is the energy of the incident photon, W_b is the binding energy of the electron (this energy must be delivered to eject an electron from an atom), m is the electron mass, v is its velocity, and the term $\frac{1}{2}mv^2$ is the kinetic energy of the ejected electron. When the ejected electron has no or only very small kinetic energy, i.e. $hf_0 = W_b$, f_0 is called the threshold (or limit) frequency. When $f > f_0$ the electrons are liberated from atoms. See Fig. 34.

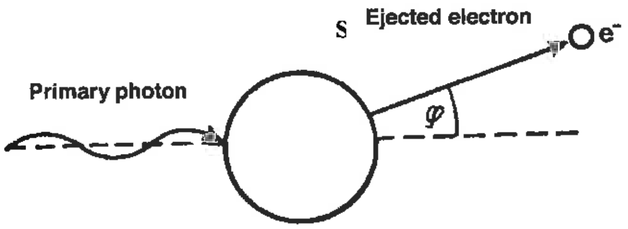

Fig. 34. The photoelectric phenomenon

Consequence:
The energy carried by the ejected electrons does not depend on the intensity of the incident radiation (light) but on the energy (i.e. on the frequency and/or wavelength) of individual incident photons.

Compton scatter:

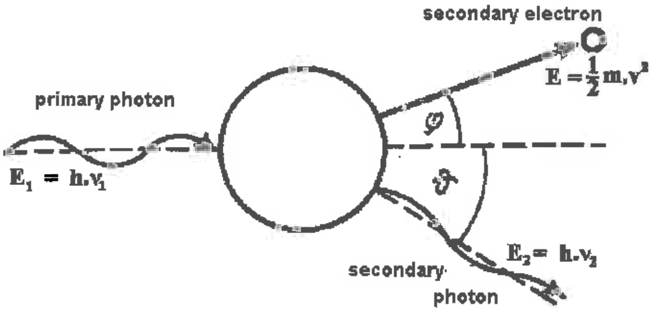

Fig. 35. Compton scatter. The Greek letter ν (nu) is used for frequency instead of *f*.

This is a phenomenon related to the photoelectric effect, occurring at higher energies of incident photons. The photon energy is not fully transformed into the kinetic energy of ejected electron and the energy necessary for ejection of the electron (binding energy W_b) so that a secondary photon of lower energy appears. Moreover, this binding energy is (in comparison with the energy of the secondary photon and the kinetic energy of electron) is so small that it can be neglected. The secondary photon does not travel in the same direction as the primary photon – it is scattered. So we can write:

$$hf = (W_b) + \frac{1}{2}mv^2 + hf'$$

where hf' is the energy of the secondary photon ($f' < f$). See Fig. 35.

Momentum of a photon:
A photon is a discrete amount of energy connected with oscillations of an electromagnetic field. Photons behave sometimes as particles with zero rest mass. They have even their own momentum which is given by the following equations:

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \qquad \left(\lambda = \frac{c}{f}\right)$$

This photon momentum manifests clearly photons obey the Law of momentum conservation.

11.3 The atomic nucleus

Composition of the atomic nucleus:
The particles forming the atomic nucleus – neutrons and protons – are called nucleons. They are attracted by the so-called strong interaction, which belongs among the four fundamental physical interactions (forces – gravitational, electromagnetic, weak and strong).
Nuclei are characterised by several main parameters:
Atomic (or **proton**) **number** *Z* – number of protons inside a nucleus.
Neutron number *N* – number of neutrons inside a nucleus.
Atomic mass (or **nucleon**) **number** *A* – number of nucleons inside a nucleus. It is evident that

$$A = Z + N.$$

The general symbol for a nucleus should be written in following form:

$${}_{Z}^{A}X \text{ (often only } {}^{A}X\text{)},$$

where X is the chemical symbol of an element.
The electric charge of a nucleus is given by the number of its protons:

$$Q_{nuc} = Ze,$$

where e is the elementary charge (electric charge of one electron or proton, 1.602×10^{-19} C).

An **element** is a substance the nuclei of which are made up of the same number of protons (number Z is the same in all the nuclei).
A **nuclide** is a substance the nuclei of which are of identical composition (numbers Z and N have the same values in all the nuclei).
Isotopes are nuclides of an element the nuclei of which have the same number Z but they differ in number of neutrons. Different isotopes of a chemical element cannot be distinguished chemically because they possess identical electron shells. They are of different mass, of course. Individual isotopes are denoted sometimes by the name of the chemical element followed by the respective nucleon number A. For example: carbon-12, carbon-14, uranium-235, uranium-238 etc.

Nuclear binding energy:
The nucleons are held together by the "strong interaction". The binding energy E_{nuc} can be defined as the energy which would be just necessary to disintegrate the nucleus into individual nucleons. During synthesis of nuclei from individual nucleons the same amount of energy is liberated.
The greater the binding energy the more stable is the nucleus. This energy can be calculated from the difference Δm between the mass of the nucleus m_{nuc}, and the sum of masses of the number of individual (free) protons and neutrons (m_p, m_n) which are contained in the respective nucleus.

$$\Delta m = Zm_p + Nm_n - m_{nuc}$$

Now it is possible to use Einstein's formula of **mass – energy equivalence:**

$$E_b = \Delta mc^2,$$

where E_b is the binding energy and Δm is the above defined mass difference (sometimes called mass excess).

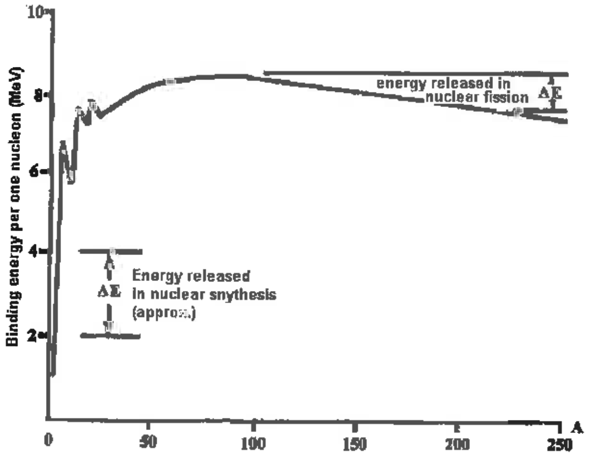

Fig. 36. Curve of the nuclear binding energy E_b per one nucleon. A – atomic mass number.

The nuclear binding energy per nucleon has different values for different nuclides – Fig. 36. Maximum binding energy per nucleon (i.e. the highest stability) can be found in nuclei which are in the middle part pf the periodic table of elements. The nucleus of the isotope $^{56}_{26}Fe$ (iron-56) possesses the greatest value of binding energy; it means that this nucleus is the most stable. Very light and very heavy nuclei are not so stable which is a fact of extraordinary importance: during synthesis of light elements (thermonuclear synthesis or fusion) and splitting of heavy nuclei (nuclear fission) considerable amount of nuclear energy are liberated.

Examples:

(thermonuclear synthesis of helium-3)

$$^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He + \gamma$$

energy liberated: 5.49 MeV

(nuclear fission of uranium-235 induced by neutron)

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3\left(^{1}_{0}n\right)$$

energy liberated: about 200 MeV

Nuclear reactor:

A **nuclear reactor** is a complex device in which a controlled nuclear fission takes place. Numerous rods made of uranium enriched by the isotope $^{235}_{92}U$ are placed in a large steel container. A **chain reaction** is started if a neutron interacts at low velocity (a moderated or thermal neutron) with a nucleus of the isotope uranium-235, and causes its fission. The two or three neutrons which are liberated by fission are slowed down (decelerated) to be able to cause new fission. The deceleration of neutrons is achieved by a **moderator** (water, graphite or another substance composed of light nuclei). The fine regulation of neutron flux in the reactor is ensured by insertion of control rods made of cadmium, for example. These control rods reduce the number of neutrons able to induce fission. The large amount of heat produced is utilized for heating water or other suitable liquid in the primary loop of the reactor. This very hot and more or less radioactive liquid warms up the water in the secondary loop, which drives a powerful steam turbine to produce electric energy. The neutron flux produced by a nuclear reactor can be also used for production of artificial radioactive elements – radionuclides.

Natural and artificial radioactivity:

Radioactivity is the ability of a nucleus to emit particles or quanta of energy. This process is also called radioactive decay or disintegration.

Radioactive nuclides are called radionuclides (more than 1600 radionuclides have been identified in nature or prepared artificially).

The radioactivity is accompanied by nuclear transformation. It means that during the radioactive decay new (daughter) nuclei arise.

We can distinguish several main types of radioactive decay according to the kind of ionizing particles produced:

α (alpha)-radiation

Helium nuclei are emitted from a radionuclide. These particles are electrically charged (+2*e* charge) and relatively heavy. They cause direct ionization of ambient atoms, and all their energy is transmitted to the medium along a very short track. It means that this radiation is only little penetrating.

Scheme:

$${}^{Z}_{A}X \rightarrow {}^{Z-4}_{A-2}X + {}^{4}_{2}He$$

Example:

$${}^{226}_{88}Ra\ (radium) \rightarrow {}^{222}_{86}Rn\ (radon) + {}^{4}_{2}He$$

β^- (beta-minus)-radiation

Electrons are emitted from a radionuclide. One nuclear neutron is transformed into a proton, an electron and a very light particle called “electron antineutrino” (not shown in the scheme and reaction below).

Interaction of these particles with matter was described in the paragraph dealing with the origin of X-rays. The ionization ability of β-particles is relatively low hence their penetration into matter is easier in comparison with α-particles.

Scheme:

$${}^{Z}_{A}X \rightarrow {}^{Z}_{A+1}X + e^-$$

Example:

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^-$$

β^+ (beta-plus)-radiation

Positrons are emitted from radionuclides. The positron is an antiparticle of the electron. One nuclear proton is transformed into a neutron, a positron and a very light particle called “positron neutrino” (not shown in the scheme and reaction below).

Scheme:

$${}^{Z}_{A}X \rightarrow {}^{Z}_{A-1}X + e^+$$

Example:

$${}^{30}_{15}P \rightarrow {}^{30}_{14}Si + e^+$$

γ (gamma)-radiation

These high energy photons are emitted from a radionuclide. A nucleus possessing some excess energy [*] emits a photon of electromagnetic radiation. The γ-emission often accompanies other types of radioactive decay. The ionization ability of this radiation is relatively very low hence it penetrates very deeply into matter. For interactions with matter, see the paragraphs dealing with the photoelectric effect and Compton scatter.

Scheme:

$${}^{Z}_{A}X^* \rightarrow {}^{Z}_{A}X + \gamma$$

Example:

$${}^{99}_{43}Tc^* \rightarrow {}^{99}_{43}Tc\ (technetium) + \gamma$$

Neutron radiation

A neutron is emitted from a radionuclide or can be liberated after collision of two accelerated nuclei.

Example:

$$ {}^{4}_{2}He + {}^{27}_{13}Al \rightarrow {}^{30}_{15}P + {}^{1}_{0}n $$

This nuclear reaction, followed by reaction ${}^{30}_{15}P \rightarrow {}^{30}_{14}Si + e^{+}$, represents the first case of artificial radioactivity observed by Frédéric and Irène Joliot-Curie in 1934.
The neutron ionizes matter indirectly. Fast neutrons transmit their energy by impacts. This process is highly effective in collisions with some light nuclei (such as hydrogen nucleus). Moderated (slow, thermal) neutrons can enter inside some heavy nuclei (such as uranium nucleus) and cause their disintegration – fission (see above).

Proton radiation
Protons are emitted from a radionuclide, or liberated after collision of two accelerated nuclei.
Example:

$$ {}^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + p\left({}^{1}_{1}H\right) $$

This nuclear reaction was the first case of artificial nuclear transmutation (transformation). It was performed by Rutherford in 1919. Nitrogen was bombarded by α-particles.

All kinds of nuclear radiation, together with X-rays, are called **ionizing radiation** because they are able to ionize atoms along their tracks (trajectories) through matter. Chemical changes of various substances or even damage to biological systems can be caused in this way.

Radioactive transmutation law – decay law:
Radioactivity is a stochastic (probabilistic) process which can be described in terms of probability. However, in great assemblies of radioactive nuclei it can be also described by simple equations:

$$ A_t = A_0 e^{-\lambda t} \qquad \text{or} \qquad N_t = N_0 e^{-\lambda t}, $$

where A_t is the **activity** (see below) of a radioactive sample at time t = t, A_0 is the initial activity of the sample at time $t = 0$, N_t is the number of nuclei left at time t = t, and the value N_0 is the initial number of radioactive nuclei at the time $t = 0$. λ is the **decay** (or disintegration) **constant** [s^{-1}] and t is the time of observation (i.e. the time during which the individual radioactive decays were counted).
Activity A of a radioactive emitter expresses the number of decays (transmutations) in it per 1 second.

1 decay per 1 s = 1 Bq (becquerel) [s^{-1}]

Half-life (time) T of a radionuclide is a quantity used for characterization of the decay rate; it is better understandable then the activity itself. It is the time during which the number of radioactive nuclei (or the activity of a sample) decreases to one half of the initial value. It is possible to derive (substitute $N_t = N_T = N_0/2$ and $t = T$ in the above formula) that

$$ T = \frac{\ln 2}{\lambda} \qquad [s], $$

where λ is the decay constant.

Main applications of ionising radiation and radionuclides:

- Attenuation of X-rays is utilized in medical imaging (radiography and CT - computerized tomography) to show inner structures of our body. γ-radiation is used for this purpose in some other imaging methods.

- The strong biological effects of ionizing radiation (including killing of tumour cells) is used in radiotherapy,
- Heat production during nuclear fission is used for production of electricity in the nuclear reactors.
- Chemical compounds (metabolites) labelled by radioisotopes make it possible to follow metabolic pathways in living organisms – these methods are called "tracing" techniques.
- Measurement of the content of various radionuclides is often used for dating of different organic and inorganic samples (e.g., radiocarbon method used in archaeology.

Accelerators:
Not only natural or artificial radionuclides and X-ray tubes can be used as sources of ionizing radiation. Intense high-energy beams of radiation can be obtained by means of **accelerators.** These devices are used for acceleration of electrically charged particles using electrostatic or electromagnetic fields.

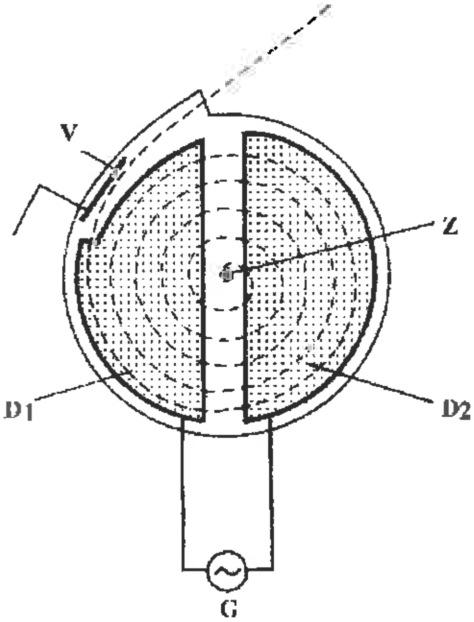

Fig. 37. The cyclotron. Z is a source of the particles (protons) to be accelerated. D_1 and D_2 – duants or dees – are metallic hollow halves of a disc. The protons are accelerated only in the gap between the duants which are connected to a generator (G) of high-frequency voltage. Otherwise, the protons are circling (spiralling) outwards within the duants where the intensity of the electric field is zero. The protons are kept on a circular (spiral) path by a magnetic field which is produced by a strong electromagnet. The magnetic force lines would be perpendicular to the plane of the picture. The most accelerated protons are deflected by a special electrode (V) and travel outside the cyclotron.

The **betatron** is used to accelerate electrons. Betatrons were commonly used to produce X-rays of high energy (see the paragraph dealing with origin of X-rays – bremsstrahlung).
The **cyclotron** (Fig. 37) is used for acceleration of particles heavier than electrons (protons, nuclei of heavy hydrogen – deuterons, small ions etc.). It can be used for direct irradiation of cancer patients but their main purpose is the production of artificial radionuclides.
High-frequency linear accelerators are used to accelerate electrons by means of microwaves. The accelerated electrons serve for production of high-energy X-rays (bremsstrahlung). They are the most often utilized accelerators in contemporary cancer treatment.
The most powerful accelerators (synchrotrons, colliders) are used in basic scientific research when studying the fine structure of matter.

11.4 Detection and measurement of ionizing radiation

In this small final chapter we will briefly mention some of the most important methods and detectors used for simple detection, as well as, precise measurement of ionizing radiation. All of these methods are based on the ability of radiation to ionize or to form free radicals and hence chemical changes in a medium.

The **Geiger-Müller counter** is one of the oldest devices used for measurement of ionizing radiation, and a good example of detectors based on ionization of gases. Its measuring probe consists of a metallic cylinder at the axis of which is placed a thin central wire. There is a special mixture of gases between the central wire and the inner surface of the cylinder. There is also a voltage difference of about 1000 V between the cylinder and the wire, which causes a high intensity electric field near the wire. Particles of radiation cause ionization of the molecules of the gas mixture. Consequently, the ions formed are attracted to the negatively charged wire. Therefore, the originally neutral molecules of the gases are ionized by impacts of the ions – an avalanche effect arises. This "avalanche" of charged particles strikes the central wire, and can be detected as an electric pulse. The impulses representing the individual particles of ionizing radiation are counted by an electronic device.

The **scintillation counter** (Fig. 38) consists of a **scintillator** – a crystal which produces visible light after irradiation by ionizing radiation – and a photomultiplier. The scintillator is enclosed in a light-proof housing. One side of the housing is transparent, so that the originating photons can come to a **photomultiplier**, which measures low-intensity light.

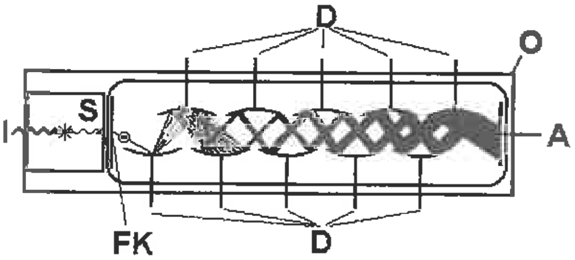

Fig. 38. The scintillation detector. I - ionising radiation represented by one photon, S - scintillator, FK - photocathode, D - dynodes, A - anode, O - light- and water-proof housing.

The photons strike the **photocathode** - a very thin layer of a metal with low electron binding energy. They eject electrons from the cathode, which are attracted and accelerated by the closest positively charged electrode, the first **dynode**. There some more electrons are ejected. These secondary electrons are attracted to the next dynode, where the process is repeated. The dynodes form an array of about ten electrodes. Resulting voltage pulses are counted in the electronic part of the instrument. The magnitude of the pulse is determined by the *energy* of the ionising particle, which can therefore be measured.

Wilson cloud chamber (or **fog trail apparatus**) is used for visualization of the trajectory of ionizing particles. These chambers are vessels filled with an oversaturated vapour. The ions formed along the particle trajectory serve as centres of condensation. Therefore, the particle trail can be seen as an array of small droplets – the fog trail. In this chamber, it is possible to follow collisions of particles with neutral atoms, their disintegration etc. The presence of a magnetic or electric field in the chamber enables us to determine the charge and velocity of the particle.

In the **bubble chamber**, a similar principle is exploited. The particles of ionizing radiation move in a vessel filled by an overheated liquid. The ions formed serve as centres of evaporation. Therefore, it is possible to observe the trajectory of the particle as an array of small bubbles.

Photographic emulsions are sensitive to the action of ionizing radiation which causes a local blackening of the emulsion. These (thick-layered) emulsions are used in scientific research to follow trajectories of ionizing particles. Blackening of the photographic film can be used also for determination of the dose of radiation absorbed by people working with sources of ionizing radiation.

Problems to solve:
29. What is the de Broglie wavelength of a proton travelling at velocity of 1, 10, 100 and 1000 km/s? (assume that the proton mass is the same as atomic mass unit)
30. Binding energy of an electron in the valence shell of an atom is 10 eV. What is the shortest wavelength of light able to cause the photoelectric effect?
31. Let us have 1 mg of the radionuclide iodine-131 the half-life of which is about 8 days. How much we will have after 80 days?

12 APPENDIX

12.1 Solutions of problems

1. 60 ms^{-2} **2.** 12.5 s **3.** 500 m **4.** 667 W **5.** 3.00238 m **6.** When increasing the displacement, the kinetic energy changes in the potential energy, meanwhile the oscillator slows down. **7.** 70 dB **8.** 0.190 m **9.** 0.0077 N **10.** 154 kPa **11.** 0.161 m^3 **12.** - **13.** The working gas receives heat from environment at low temperature and transmits heat to the environment at higher temperature. **14.** 1.8 MJ **15.** 2.78 A **16.** 104.17 μF **17.** 15197 C **18.** 2×10^{-6}m **19.** 159 Ω **20.** 0.785 V **21.** 651 V **22.** 1.5 **23.** 0.1m **24.** 1875 **25.** 0.15 mm **26.** 3600 cd **27.** 1355 s **28.** 1.34 **29.** 0.4 nm, 0.04 nm, 4 pm, 0.4 pm **30.** 123 nm **31.** 0.000488 mg

12.2 Multiple-choice test questions

The examples of the entrance exam questions for physics are of the type used for entrance exams in recent years.

Units, dimensions, decimal multiples, scalars, vectors

1. Which group contains only SI base units?
a) cm, s, J b) K, cd, s c) kg, cd, V d) s, mol, V

2. Which unit is *not* an SI base unit?
a) K b) mol c) cd d) J

3. Which of the following sets does *not* contain only base or derived SI units?
a) meter, joule, candela, Tesla
b) second, ampere, newton, mole
b) second, mole, kilogram, candela
d) candela, second, gram, joule

4. W.s can be used as the units of:
a) output b) surface tension c) efficiency d) energy

5. The units of angular momentum are:
a) kgms b) kgm^2s^{-1} c) $kgms^{-2}$ d) Nm^{-1}

6. Which of the following can be used as a unit of electrical power?
a) W/m b) V/A c) VA d) tesla

7. Which of the following pairs of units are equivalent?
a) ohm [VA] b) joule [$kgms^{-2}$] c) watt [Js] d) decibel [Wm^{-2}]

8. If one cubic meter of water is spilled over an area of one square kilometre, the layer of water will have a thickness of:
a) 1 mm b) 0.1 mm c) 1 µm d) 10 µm

9. Which set consists of equivalent units?
a) 100 nm, 0.1 µm, 1×10^4 pm, 1×10^{-7} m b) 1 m^3, 1×10^3 l, 1×10^6 mm^3, 1×10^9 mm^3
c) 1 J, 1W.s, 1×10^9 nJ, 1×10^{12} pJ d) 1 mV, 1000 µV, 0.001 V, 1×10^{12} nV

10. Which one is *not* correct?
a) 100 ml = 10^5 µl b) 1 h = 3.6×10^{15} ps
c) 100 A = 10^{11} nA d) 1 kJ = 3.6 Wh

11. Which set contains only scalar quantities?
a) distance, surface tension, pressure
b) energy, intensity of electric field, magnetic induction
c) momentum, energy, moment of a force d) magnetic induction, acceleration, momentum

12. Which one of the following quantities is a vector?
a) acceleration b) pressure c) electric voltage d) energy

Kinematics and dynamics of point masses and solid objects, gravitational field

13. A rocket fired horizontally from a stationary platform was given an acceleration of 40 $m.s^{-2}$. How long does it take to move a distance of two kilometres (assume linear movement at constant acceleration)?
a) 80 s b) 50 s c) 40 s d) 10 s

14. A man applies a force of 200 N to a cart of mass 100 kg moving over a horizontal surface. What will be the velocity of the cart after 1 second (assume a frictionless horizontal surface)?
a) 0.5 ms^{-1} b) 1.0 ms^{-1} c) 2.0 ms^{-1} d) 5.0 ms^{-1}

15. With what velocity does a body of mass 10 g falling from a height of 10 m above the surface of a planet hit the surface (assume a uniform gravitational field of 5.00 ms^{-2} and that the planet has no atmosphere)?
a) 5.00 ms^{-1} b) 10.0 ms^{-1} c) 14.1 ms^{-1} d) 105 ms^{-1}

16. Ball A moving with horizontal speed 10 ms^{-1} rolls across the edge of a chasm on the Moon. Ball B also falls into the same chasm but with zero horizontal speed. Which one of the following is correct?
a) ball A will land earlier than ball B
b) at the moment of landing the speed of ball A will be higher than that of ball B
c) ball B will land earlier than ball A
d) at the moment of landing the speed of ball B will be higher than that of ball A

17. With what velocity must a body of mass 50 kg be projected perpendicularly upwards so that it can reach a height of 1000m above the surface of a planet without atmosphere and having a uniform gravitational field of $a_g = 0.500\ ms^{-2}$?

a) 166 ms^{-1} b) 31.6 ms^{-1} c) 16.6 ms^{-1} d) 10.0 ms^{-1}

18. A body of volume 40 ml and mass 20 g is ascending from the bottom of a vessel containing water ($\rho = 1000\ kgm^{-3}$) with a constant velocity of 6 cms^{-1}. What is the total force which is opposing the motion of the object? ($a_g = 10\ ms^{-2}$)

a) can't work it out as do not know the shape of the body

b) 0.20 N c) 0.40 N d) 200 N

19. What was the initial speed of a meteorite of mass 1 kg if a total of 18 MJ of thermal and other forms of energy were produced during its fall through the atmosphere of a planet and its impact on the surface?

a) 18 000 ms^{-1} b) 1 800 ms^{-1} c) 6000 ms^{-1} d) can't decide

20. A cyclist shifted gear before going uphill. As a consequence the total work which he has to do whilst going up the hill: (ignore friction)

a) increased b) decreased c) did not change d) not sure

21. A worker pulls up a load of 0.75 t using a pulley through a height of 2 m. He works at a uniform rate of 300 W. How long will he take to pull up the load? (assume no friction, g = 10 ms^{-2})

a) 5 s b) 150 s c) 400 s d) 50 s

22. A car moves at constant velocity and its engine supplies a constant power. Which of the following is correct (assume the force produced by the engine is in the same direction as that of the motion)?

a) $P = Fa$ b) $P = F/v$ c) $P = Fv$ d) $W = P$

23. A wheel of diameter 50 cm is rotating such that a point on its perimeter moves with a speed of 20 ms^{-1}. How many revolutions does the wheel turn per minute?

a) $2400/\pi$ min^{-1} b) $1200/\pi$ min^{-1} c) 2400 min^{-1} d) 1200 min^{-1}

24. What is the angular velocity of a body rotating with a uniform motion if it takes a tenth of a second to complete one revolution?

a) 0.1 Hz b) 10 Hz c) 20π $rads^{-1}$ d) 10π $rads^{-1}$

25. What is the kinetic energy of a small body of mass 10g moving in a circle of radius 1 m with an angular velocity of 10 $rads^{-1}$?

a) 0.1 J b) 1 J c) 0.05 J d) 5 J

26. An object of mass 1 kg is whirled round in a circle at the end of a string at 10 revolutions per second. If the radius of the circle is 10 m, what tension in the string is required to keep the body moving in its circular path? (neglect any effect of gravity)

a) 3.25 kN b) 4.00 kN c) 12.5 kN d) 19.7 kN

27. Two point masses originally 1m apart are moved such that they are separated by a distance of 3 m. The gravitational force between the two masses:

a) did not change b) became a half c) became a third d) became one ninth

Hydrostatics and hydrodynamics

28. The fact that bubbles of gas always move upwards towards a water surface can be explained using:
a) Bernoulli's equation b) Archimedes's law c) Pascal's law d) continuity equation

29. The total pressure at a depth of 10 m below sea level is approximately equal to:
a) half the atmospheric pressure b) equal to the atmospheric pressure
c) double the atmospheric pressure d) ten times the atmospheric pressure

30. The unit 1 mm of mercury or one torr is can be expressed in SI units as:
a) 0.001 m b) 1 N c) 1 Pa d) 1000 hP

31. Bubbles are ascending up to the surface of a liquid. The upward force on them (think of the effect of the hydrostatic pressure on the bubbles) will be:
a) decreasing b) zero c) constant d) increasing

32. The upward force on a body which is totally submerged will be bigger:
a) in the case of a ball with diameter 1m than in the case of a cube of edge 1 m.
b) in the case of a ball than a cube with the same volume
c) in the case of a cube than a ball with the same volume
d) in the case of a cube of edge 1m than a ball with diameter 1 m

33. Bernoulli's equation is a special case of the law of:
a) conservation of energy b) conservation of mass c) Pascal's law d) conservation of momentum

34. The term $\frac{1}{2}\rho v^2$ in Bernoulli's equation has the same dimensions as:
a) pressure b) energy c) power d) volume

35. Water is passing through a pipe of cross-section 0.5 m^2 with a speed of 5 ms^{-1}. Consider water as an ideal liquid with density 1000 kgm^{-3}. What is the kinetic energy of 1m^3 of water?
a) 5 kJ b) 12.5 kJ c) 25 kJ d) there is not enough data for the calculation

36. Water passing through a pipeline enters a section of pipe with a smaller radius. If its speed increases four times the original value what is the ratio of the higher to the lower radius of the pipe?
a) 1:4 b) 4:1 c) 1:2 d) 2:1

37. An ideal liquid flows out of a short flexible horizontal tube at the bottom of a vessel. If we turn the tube upwards and we neglect the friction, the liquid spurts upwards to a level:
a) higher than the level of liquid in the vessel
b) equal to that of the level of liquid in the vessel
c) to the height of the level of liquid in the vessel divided by the square root of two
d) equal to half that of the level of liquid in the vessel

Oscillations and acoustics

38. The total mechanical energy of an undamped mechanical oscillator is:
a) the highest at maximum displacement b) the lowest at minimum displacement
c) constant d) zero at minimum displacement

39. A body vibrating with harmonic motion has maximum speed at the moment when:
a) it reaches maximum acceleration b) it reaches the maximum value of potential energy
c) the value of its displacement is just $0,5y_{max}$ (y_{max} is the amplitude)
d) the value of its displacement is $\sqrt{2}\ y_{max}$

40. An undamped harmonic oscillator reaches maximum potential energy:
a) at maximum displacement b) at zero displacement
c) when the potential energy of the oscillator is constant
d) only at the moment when it is put into motion

41. The equation $y = y_m \sin(\omega t + \phi_0)$ determines:
a) only the amplitude of oscillation of a harmonic oscillator
b) the instantaneous displacement of a harmonic oscillator from its position of equilibrium
c) only the maximum displacement of a harmonic oscillator from its position of equilibrium
d) only the initial displacement of a harmonic oscillator from its position of equilibrium

42. A harmonic oscillator will return to the same displacement after a phase change of:
a) 1 rad b) $\pi/2$ rad c) π rad d) 2π rad

43. The angular frequency of a harmonic oscillator is defined as:
a) $\omega = 2\pi T$ b) $\omega = 1/f$ c) $\omega = fT$ d) $\omega = 2\pi$

44. What is the main difference between sound and ultrasound?
a) the speed of ultrasound is higher than the speed of sound
b) the sound waves are mechanical, while ultrasound waves are electromagnetic
c) the sound oscillations are transverse, whilst those of ultrasound are longitudinal
d) the frequency of sound is lower than the frequency of ultrasound

45. Watt per square meter (Wm^{-2}) is the unit of:
a) loudness of sound b) sound intensity c) acoustic power d) acoustic pressure

46. The total span of sound audibility (from the point of its" power") of a healthy human ear is around:
a) 16 - 20 000 dB b) up to 80 dB c) up to 100 dB d) up to 130 dB

47. Which of the following statements is true?
a) Ultrasound can propagate only in gases b) Sound can propagate only in a vacuum
c) Sound waves propagating in air are transverse mechanical oscillations
d) Sound oscillations are not oscillations in a magnetic field.

48. Sound has intensity 1 mWm^{-2}. What is the value of this intensity in decibel (dB)?
a) 1 dB b) 0.001 dB c) 0.002 dB d) 90 dB

49. Vowels have the character of:
a) simple tones b) higher overtones c) composite tones d) noise

Thermodynamics, changes of state, surface tension, elasticity

50. The universal gas constant has the base SI units:
a) JKmol b) $JK^{-1}mol^{-1}$ c) JK d) JK^{-1}

51. The term pV in the equation of state of an ideal gas has units?
a) J b) JK c) JK^{-1} d) $JK^{-1}.mol^{-1}$

52. The Carnot Cycle is formed:
a) by two isothermal and two adiabatic processes
b) by one isothermal and two adiabatic processes
c) by one isothermal and one adiabatic process
d) by two isochoric and two isobaric processes

53. The work done by a thermodynamic system whilst pushing a piston is proportional to the:
a) immediate volume b) temperature c) pressure d) internal energy

54. During a reversible isothermal expansion of an ideal gas its pressure:
a) increase b) decreases c) remains constant d) and volume are growing

55. The first law of thermodynamics represents:
a) a rule describing the spontaneous movement of heat from colder objects to warmer objects
b) a rule explaining the transformation of work into pressure
c) a rule describing the spontaneous movements of heat from warmer objects to colder objects
d) a special formulation of the law of conservation of energy

56. An ideal gas cannot perform volumetric work during an:
a) isothermal process b) isochoric process c) isobaric process d) adiabatic process

57. In which of the following processes will the pressure *not* decrease when the volume is increased?
a) isometric b) isobaric c) isochoric d) adiabatic

58. During an isobaric process an ideal gas follows the equation:
a) $pV = const.$ b) $VT = const.$ c) $V/(nT) = const.$ d) $nRT = const.$

59. If we double the temperature in an isochoric process the:
a) pressure doubles b) volume decreases to half
c) volume doesn't change d) pressure decreases to half

60. A refrigerator is left with its door open in a thermally insulated room. The temperature in the room will:
a) decrease b) stay constant c) increase
d) increase only in the case when the initial temperature in the room was lower than the temperature inside the fridge before opening the doors of the fridge

61. Which of the following processes is *not* an example of sublimation?
a) evaporation of solid carbon dioxide b) drying of frozen washing
c) formation of black soot by a burning candle in conditions of low air supply
d) decrease of crystalline iodine in an open vessel

62. When the pressure in an insulated vessel is lowered, the amount of ice increases compared to liquid water. The temperature:
a) does not change b) decreases c) increases d) first increases and then decreases

63. In which of the following are there more molecules than in 1 kg of water?
a) 1 kg of helium b) 1 kg of oxygen c) 1 kg of glucose d) 1 kg of air

64. The surface tension of liquids can be defined as the:
a) force perpendicular to unit length of edge of surface membrane of liquid
b) intensity of electrical tension between a liquid and its surroundings
c) force needed to tear the surface of a liquid
d) force produced per unit surface area of liquid

65. If we connect the inside of two different size soap bubbles using a hollow pipe surface tension theory predicts that the:
a) smaller one will disappear and the bigger will grow
b) volume of the bubbles will become equal
c) we will not see any change in sizes of bubbles
d) is not possible to predict the behaviour of the bubbles

66. A capillary tube put in a liquid leads to capillary depression. If a capillary of smaller cross sectional area is used there is:
a) an increased fall of liquid level in the capillary
b) a lowered fall of liquid level in the capillary
c) no change in the fall of the liquid level in the capillary
d) often a change from depression to elevation

67. A bar with a cross section of 1 cm^2 has a Young's modulus of elasticity of 100 GPa. What force is needed to increase its relative extension by one tenth of a percent?
a) 10 N b) 100 N c) 1 000 N d) 10 000 N

68. Normal stress is defined as the:
a) force applied to a unit length of an elastic bar or wire
b) pressure applied to a unit length of an elastic bar or wire
c) force applied on a unit cross-sectional area of a bar or wire
d) pressure applied on a unit cross-sectional area of a bar or wire

Electricity and magnetism

69. The electrostatic force on a unit electrical charge is numerically equal to the:
a) electric potential b) value of the electric dipole
c) dielectric constant of the surroundings d) intensity of the electric field

70. The relative permittivity of water has a value about 80. If we move two charges with the same sign from air to water their mutual repulsion will:
a) increase b) not change c) decrease
d) decrease only when there is a surplus of anions or cations in the water

71. The electrostatic force between two units of electrical charge is:
a) proportional to the distance between them
b) proportional to the permittivity of the surrounding medium
c) equal to the electrical potential
d) given by the expression $1/(4\pi\varepsilon r^2)$

72. If we move two electric charges of opposite sign apart to four times the original distance between them the force with which the bodies are attracted will:
a) decrease to 1/2 b) decrease to 1/4 c) decrease 1/16 d) will not change

73. Two capacitors of equal capacitance connected in series are connected in parallel with a third capacitor also of the same capacitance. The total capacitance of this combination of capacitors is equal to 50 nF. The capacitance of each capacitor is:
a) 16.666 nF b) 150 nF c) 25 nF d) 100 nF

74. If the plates of a capacitor are moved closer together to half the original distance. The capacitance:
a) will increase twice b) will decrease to half c) will increase by half of the original value
d) will stay constant

75. How long would it take to charge a capacitor of capacitance 1μF to a voltage of 10 V with an electrical current of 1 μA?
a) 0.1 s b) 1 s c) 10 s d) 1000 s

76. The resistance of a conductor is:
a) inversely proportional to its length and cross-sectional area
b) directly proportional to its length and cross-sectional area
c) inversely proportional to its cross-sectional area and directly proportional to its length
d) depends only on the length of the conductor, its cross-sectional area is not important

77. Three resistors of resistance 30 MΩ, 60 MΩ and 90 MΩ are connected in parallel. What is the total resistance of the system?
a) 16 MΩ b) 60 MΩ c) 180 MΩ d) 90 MΩ

78. Kirchhoff's first law deals with:
a) electrical charges in electrolytes
b) voltage induction by changes in a magnetic field
c) electrical currents at the nodes of electrical circuits
d) voltages in electrical circuits

79. As the temperature of a conductor increases we expect that:
a) its electrical resistance will decrease
b) its electrical conductance will increase
c) more heat will be released when passing the same electrical current
d) less heat will be released when passing the same electrical current

80. The force on a current carrying electrical conductor in a magnetic field would be zero if the:
a) angle between the conductor and the vector B is 0°
b) the current is alternating
c) angle between the conductor and the vector B is 90°
d) the lines of force of the magnetic field are parallel

81. Which formula can be used to calculate of impedance of a capacitor?
a) $X_C = 1/\omega C$ b) $X_C = \omega/C$ c) $X_C = \omega C$ d) $X_C = U/Q$

82. What is the voltage induced in a conductor 30 cm long, which is moving with speed 20 cms^{-1} perpendicularly to a homogeneous magnetic field of induction B = 0.2 T in a plane perpendicular to the field?
a) 12 V b) 1.2 V c) 12 mV d) 1.2 mV

83. The magnetic induction B near a straight wire carrying a direct-current is:
a) dependent on the metal from which the conductor is made
b) directly proportional to the magnitude of the current
c) directly proportional to the length of the conductor
d) equal to zero (there is no magnetic field)

84. An electrically charged particle moves into a magnetic field in a direction parallel to that of the magnetic induction B. What effect will the field have on its trajectory?
a) deflect it according to the right hand rule b) deflect it according to the left hand rule
c) it will move more slowly d) it will move faster

85. The magnitude of the magnetic field inside a very long cylindrical coil with electrical current can be increased by:
a) lowering the number of turns
b) lowering the number of turns per unit length of coil
c) inserting a material with high permeability into the coil
d) reducing the radius of coil whilst keeping the number of turns per unit length of coil constant

86. The luminous figures on the screen of a CRT are produced by:
a) the movement of the point of incidence of a narrow luminous beam
b) by projection of an image which is formed by a photomultiplier situated at the back of the screen
c) by movement of the point of incidence of a beam of electrons
d) by a corona discharge

Optics

87. Which one of the following statements about the refraction of light is true?
a) the angle of refraction is the same as the angle of incidence
b) the angle of refraction is always bigger than the angle of incidence
c) the angle of refraction is always smaller than the angle of incidence
d) the sum of the angles of refraction and incidence is always 90°.

88. When a ray of light moves from vacuum to glass:
a) there will be refraction towards the normal
b) there will be refraction away from the normal
c) refraction will not occur
d) total internal reflection will occur

89. Light rays coming from infinity and moving in a direction parallel to the principal axis of a thin converging lens intersect 25 cm behind it. The Dioptric power of the lens is:
a) -0.04 D b) +0.04 D c) +25 D d) +0.25

90. The image created by a lens is upright and virtual. The lens:
a) cannot be a diverging lens b) cannot be a converging lens
c) can be only a diverging lens d) can be both a diverging as well as a converging lens

91. Which colour of light has the longest wavelength?
a) green b) blue c) red d) violet

92. The wavelength of ultraviolet light is:
a) shorter than approximately 400 nm b) in the range 400 - 800 nm
c) longer than approximately 800 nm d) longer than approximately 800µm

93. Which of the following is caused by the interference of light?
a) shaking of the air above a candle
b) mirage
c) dispersion of polychromatic light in a prism
d) colourful stains in water polluted by a small amount of oil

94. At what vertical distance from an illuminated surface does a 900 cd bulb produce an illumination of 100 lux?
a) 9 m b) 10 cm c) 0.3m d) 3 m

Elementary particles, nuclear and quantum physics

95. Which of the following particles is not deflected in its motion by an electrical field?
a) β-particle b) neutrino c) proton d) α–particle

96. X-rays are:
a) a stream of fast electrons
b) a stream of fast ions
c) electromagnetic radiation with wavelength over 10 nm
d) electromagnetic radiation with wave length under 10 nm

97. The isotopes of uranium are different from each other in the:
a) number of electrons b) number of protons
c) state of matter d) number of neutrons in the nucleus

98. A radionuclide has a half-life 4 days. Its activity after 16 days decreases from its original value to:
a) 1/4 b) 1/8 c) 1/16 d) zero

99. Which one of the following radiations doesn't originate in the nucleus of an atom?
a) α-radiation b) β-radiation c) x-radiation d) γ-radiation

100. The basic principle of the laser is:

a) Einstein's monochromatic effect
b) collimation of light in one direction
c) Stimulated emission of light
d) spontaneous emission of light

Correct answers ("e" means "no answer is correct")
1b, 2d, 3d, 4d, 5b, 6c, 7e, 8c, 9c, 10d, 11a, 12a, 13d, 14c, 15b, 16b, 17b, 18c, 19c, 20c, 21d, 22c, 23a, 24c, 25e, 26e, 27d, 28b, 29c, 30e, 31d, 32d, 33a, 34a, 35b, 36d, 37b, 38c, 39e, 40a, 41b, 42d, 43e, 44d, 45b, 46d, 47d, 48d, 49c, 50b, 51a, 52a, 53c, 54b, 55d, 56b, 57b, 58c, 59a, 60c, 61c, 62c, 63a, 64a, 65a, 66a, 67d, 68c, 69d, 70c, 71d, 72c, 73e, 74a, 75c, 76c, 77a, 78c, 79c, 80a, 81a, 82c, 83b, 84e, 85c, 86c, 87e, 88a, 89e, 90d, 91c, 92a, 93d, 94d, 95b, 96d, 97d, 98c, 99c, 100c

12.3 Reading numerical expressions

1.451	seven point four five one
6.023×10^{23}	six point zero two three times (multiplied by) ten to the twenty-third (to the power of twenty three)
$\frac{1}{4}$	one quarter (one fourth)
$\frac{1}{60}$	one over sixty (one sixtieth)
$\frac{2}{3451}$	two over three thousand four hundred and fifty-one
2^2	two squared
2^{-2}	two to the minus two (to the power of minus two)
2^3	two cubed
2^{16}	two to the sixteenth (to the power of sixteen)
$\sqrt{16}$	square root of sixteen
$\sqrt[3]{8}$	cube root of eight

12.4 Mathematical operators and symbols

+	plus (and)
-	minus (less, from)
×, *, .	times, multiplied by
÷, /	divided by (over)
>	larger than, greater than
<	smaller than, less than
=	equals, is equal to

≠	is not equal to
≥	larger (greater) than or equal to
≤	smaller (less) than or equal to
( )	(round) brackets, parentheses
[ ]	square brackets
{ }	braces
*	asterisk
“...”	quotation marks
‘...	apostrophe
,...	comma
.	full stop (American period)
:	colon
;	semicolon
-	hyphen
_	dash
/	slash
\	backslash
____________	full line
....	dotted line
- - - - - - - -	dashed line
-·-·-·-·-·-·-	dash and dot line

12.5 Mathematical expressions

$C = A + B$

Capital C equals capital A plus capital B. Capital C is the sum of capital A and capital B, i.e. the result of **addition** (capital A and capital B are added).

$C = A - B$

Capital C equals capital A minus capital B. Capital C is the difference of capital A and capital B, i.e. the result of **subtraction** (capital B is subtracted from capital A).

$c = ab\ (a \times b)$

C equals a multiplied by (times) b. C is the product of a and b, i.e. the result of **multiplication** (a is multiplied by b).

$c = a \div b\ (a/b)$

C equals a divided by (over) b. C is the quotient of a and b, i.e. the result of **division** (a is divided by b)

$C = A^b$

Capital C equals capital A to the power of b. Capital C is the b-*th* power of capital A. Capital A raised to the power of b.

$C = \sqrt[b]{A}$

Capital C is the b-*th* root of capital A.

$y = log\, x$
Y is equal to the (base 10) logarithm of *x*.

$y = ln\, x$
Y is equal to the natural logarithm of *x*.

$y = log_2\, x$
Y is the logarithm of *x* to base 2.

$$Z = \frac{dy}{dx}$$

Capital *Z* equals the derivative of *y* with respect to *x*; capital *Z* is the result of **differentiation**.

$$Z = \frac{\partial y}{\partial x}$$

Capital *Z* equals the partial derivative of *y* with respect to *x*.

$$A = \int_{S_1}^{S_2} F(S).dS$$

Capital *A* equals the integral of function capital *F* (with respect to capital *S*) from S_1 to S_2, capital *A* is the result of **integrating**.

$$A = \sum_{i=1}^{n} x_i$$

Capital *A* is the sum of *x* (sub) *i* for *i* equal to 1 to *n*.

$$\lim_{b \to 0} \frac{a+b}{b}$$

Limit of (the term) *a* plus *b* over *b* for *b* converging to zero.

Δx	delta *x*
k', k''	*k* prime, *k* double prime
$\bar{x}$	*x* bar
x_i	*x* sub *i*
$n!$	*n* factorial
$\sin \alpha$	sine *alpha*
$\cos \beta$	cosine *beta*
$\tan \gamma$	tangent *gamma*
$\mathrm{cotan}\, \delta$	cotangent *delta*

12.6 Reading some formulas

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad \text{(“impedance”)}$$

Capital *Z* equals the square root of (the expression) capital *R* squared plus *omega* capital *L* minus one over *omega* capital *C* all squared.

$T = \frac{\ln 2}{\lambda}$ ("half-life time")

Capital *T* equals the natural logarithm of two over *lambda*.

$A_t = A_0 e^{-\lambda t}$ ("decay law")

Capital *A* sub *t* equals capital *A* sub nought times *e* to minus *lambda t*.

$\mathrm{pH} = -\log a_H$ ("acidity")

pH equals minus (base 10) logarithm of *a* sub capital *H*.

$R = Ae^{-\frac{B}{T}}$ ("resistance of a thermistor")

Capital *R* equals capital *A* times *e* to minus capital *B* over capital *T*.

$\tau = \eta \frac{\partial v}{\partial x}$ ("shear stress of a fluid")

Tau equals *eta* times partial derivative of *v* with respect to *x*.

OVERVIEW OF PHYSICS

Minimum knowledge required for entrance test – a guide to English physical terminology

Compiled and Edited by
Vojtěch Mornstein, M.D.

Published by Masaryk University, Brno, 2010
Second revised edition
Printed by Tiskárna Helbich, a. s., Valchařská 36, 614 00 Brno

ISBN 978-80-210-5192-8
ISBN 80-210-1560-8 (1. vyd.)